ADVANCED MACROECONOMICS

Tutorial

Recommended by the Methodical Commission of the Institute of Economics and Entrepreneurship, studying at the B.Sc. Programme 38.03.01 “Economics” in English

Nizhni Novgorod
2017
МАКРОЭКОНОМИКА
(ПРОДВИНУТЫЙ УРОВЕНЬ)

Учебно-методическое пособие

Рекомендовано методической комиссией Института экономики и предпринимательства ННГУ для иностранных студентов, обучающихся по направлению подготовки 38.03.01 «Экономика» (бакалавриат) на английском языке

Нижний Новгород
2017

Рецензент: д.э.н., профессор М.Л. Горбунова

В настоящем пособии изложены учебно-методические материалы по курсу «Макроэкономика» для иностранных студентов, обучающихся в ННГУ по направлению подготовки 38.00.01 «Экономика» (бакалавриат).
Пособие включает 8 базовых единиц курса, для каждой из которых приведены основные понятия, принципы и модели, практические задания. В приложении приведен перечень основных эффектов и проблем макроэкономики, а также словарь терминов. Пособие завершает список рекомендуемой литературы.

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**RECOMMENDED LITERATURE**

**APPENDIX**
Unit 1. The alternative approaches in macroeconomics: a comparative analysis

Differences between Keynesian and Neoclassical approaches

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Unit 2. The main macroeconomic identities and models in a closed economy

2.1. Main notions, concepts and effects of macroeconomics

Aggregate sectors of the national economy:
Householders, Firms, Government, and Foreign Sector (the Rest of the World)

Main aggregate variables:

1. Gross domestic product (Y):
   - nominal GDP \( Y_n = \sum_{i=1}^{m} p_i \times q_i \) and real GDP \( Y_r = \sum_{i=1}^{m} p_{i0} \times q_i \); deflator of GDP:
     \[ (\text{Def}_{\text{GDP}} = \frac{Y_n}{Y_r}) \]; economic growth rate:
     \[ g_t = \left( \frac{Y_{R_t}}{Y_{R_t-1}} - 1 \right) \times 100\% . \]
   - actual GDP \( Y \) and potential GDP \( Y_f \); output gap \( \frac{Y - Y_f}{Y_f} \).

2. Price level and inflation rate.
   - deflator: \( \text{Def}_{\text{GDP}} = \frac{\sum p_t \times q_t}{\sum p_0 \times q_t} \); \( \pi_t = \left( \frac{\text{Def}_{t-1}}{\text{Def}_{t}} - 1 \right) \times 100\% . \)
   - consumer price index: \( \text{CPI} = \frac{\sum p_t \times q_t}{\sum p_0 \times q_t} = \sum p_{t0} \times \eta_0 , \quad \eta_0 = \frac{p_0 \times q_0}{\sum p_0 \times q_0} \);
     \[ \pi_t = \left( \frac{\text{CPI}_{t-1}}{\text{CPI}_t} - 1 \right) \times 100\% . \]

3. Employment and unemployment. Actual level of unemployment \( u \) and natural level of unemployment \( u_n \). Okun’s law: \( \frac{Y - Y_f}{Y_f} = \gamma \times (u - u_n) . \)


Main regulative policies:
Fiscal, Monetary, Exchange Rate, and Foreign Trade Policy.

Purposes of the policies: 1) economic growth; 2) full employment; 3) prices stability; 4) stability of the balance of payments («the magic quadrangle» after Jan Tinbergen).

Short-run and long-run periods of economic performance

Short-run is the period of time when:
   - price level rigidity (non-elasticity) takes place;
   - money supply affects the real economic variables;
   - the economy does not adapt to the shocks completely;
   - the actual GDP may deviate from its potential level, and the actual unemployment does not coincide with its natural level.

Long-run is the period of time when:
   - price level is perfectly flexible;
   - neutrality of the money takes place, that is, changes in money supply do not affect the real economic variables;
   - the economy adapts to the shocks completely;
   - the actual GDP is equal to the potential GDP, and the actual level of unemployment is equal to its natural level.
2.2. Main macroeconomic identities

1. Formation and distribution of gross national income: \[ Y = C + I_p + G + NX \] where \( C \) – consumption; \( I_p \) – gross private internal investment; \( G \) – government purchases of final goods and services; \( NX \) – net export (export \( X \) – import \( Z \)).

2. Formation and distribution of disposable income: \[ Y_d = Y - T + TR + N \] – formation of gross income at the disposal of the home private sector, where \( T \) – taxes; \( TR \) – transfers from government to private sector; \( N \) – paid interests for public bonds held by private sector. \( Y_d = C + S_p \) – distribution of disposable income, where \( S_p \) – private savings.


4. Distribution of private savings: \[ S_p = I_p + BD + NX \] where \( BD \) – public budget deficit \( (BD = G + TR + N - T) \).

5. Balance of payments: \( NX = I_r + \Delta R \), where \( NX \) – trade account, \( I_r \) – capital outflows abroad, i.e. domestic country’s foreign investment (capital account of the balance of payments with an opposite sign), \( \Delta R = \Delta R_p + \Delta R_g \) – changes in the country’s international reserves, both private \((\Delta R_p)\) and official ones \((\Delta R_g)\).

6. State budget balance: \( BD = \Delta M_{bd} + \Delta B \). Budget deficit is financed by credits to government from the Central Bank and thus by money emission \((\Delta M_{bd})\) and by government offering of public bonds \((\Delta B = \Delta B_p + \Delta B_g)\), that are finally purchased by the private sector \((\Delta B_p)\) and by the Central Bank \((\Delta B_g)\).

7. Three channels of money supply: \( \Delta M = \Delta M_1 + \Delta M_2 + \Delta M_3 \), \( \Delta M_1 \) – Central Bank gets credits to the national economy: commercial banks and government («credit channel»): \( \Delta M_1 = \Delta M_{cb} + \Delta M_{pd} \) (in stationary economy \( \Delta M_{cb} = 0 \)); \( \Delta M_2 \) – central bank purchases the public bonds («stock channel») to finance the part of budget deficit \((\Delta M_2 = \Delta B_g)\); \( \Delta M_3 \) – central bank purchases foreign currency («exchange channel») and replenishes the official reserves \((\Delta M_3 = \Delta R_g)\).

8. Finally, private savings are distributed in the forms: \[ S_p = I_p + I_r + \Delta M + \Delta B_p + \Delta R_p \]

9. General rule for savings and investment: \[ S_p + S_g + S_r = I_p + I_g + I_r \] Summary savings from all the sectors of economy are equal to their summary investment.

2.3. The equilibrium macroeconomic models

Macroeconomic models differ:
1. Exogenous and endogenous variables (inputs and outputs).
2. Some models are based on perfect mobility of the resources (mainly the capital), and other assume non-perfect mobility.
3. Statics, Comparative Statics and Dynamics.
2.3.1. Deriving the aggregate demand function for the goods market (AD) based on the short-run IS-LM model

The IS-LM model proposed by John Hicks and P. Samuelson describes the closed economy. In the simple two-sector model, including only household and firms sectors, aggregate demand on the goods market is forming on the basis of co-equilibrium on two other markets:

1) capital market: \( S(Y) = I(r) \). Savings, depending on the level of real income \( Y \), equals to investment, depending on real interest rate \( r \);

2) money market: \( L(Y, r) = M_s / P \). The sum of transactions liquidity demand, depending on real income, and assets liquidity demand, depending on real interest rate, equals to real money supply \( (M_s / P) \).

In the extended three-sector model, which adds the public sector, the first mentioned equation is transformed into the balance of withdrawals (leakages) and injections in the national income: \( S(Y) + T = I(r) + G \), where \( T = T_a + t \cdot Y \) – total tax payments, comprising autonomous tax payments \( T_a \) \(^1\) and income based taxes \((t – income tax rate)\); \( G \) – government purchases of goods and services.

1. The equilibrium on capital and goods markets. Deriving the IS curve

The total expenditures in the closed economy include consumption, investment and government purchases of goods and services. And the statement of equilibrium implies that they are equal to the total income: \( C + I + G = Y \).

A. The consumption function: \( C = C_a + c_y \cdot Y_d \), where \( C_a \) – autonomous consumption, \( c_y \) – marginal propensity to consume as to disposable income \( (Y_d) \). The disposable income is calculated with using the formula: \( Y_d = Y - T + TR_a \), where \( T = T_a + t \cdot Y \) – taxes, \( TR_a \) – autonomous transfers.

B. The investment function: \( I = \eta \cdot Y - \beta \cdot r \), where \( \eta \) – marginal propensity to invest, \( \beta \) – sensitivity of investment with respect to real interest rate.

B. The function of government purchases (the state procurement function): \( G = G_a \), where \( G_a \) – autonomous part of it.

In the model considered we don’t take into account the transfers and government purchases dependency on the level of income, but later this assumption we will be get over.

After summing three functions and some alterations we can derive the following equation for the total expenditures in economy: \( Y = \frac{A_a - \beta \cdot r}{1 - (c_y \cdot (1 - t) + \eta)} = (A_a - \beta \cdot r) \cdot m_a \), where \( A_a = C_a - c_y \cdot T_a + c_y \cdot TR_a + G_a \) – total autonomous expenditures; \( m_a = \frac{1}{1 - (c_y \cdot (1 - t) + \eta)} \) – multiplier of autonomous expenditures.

The IS function demonstrates the simultaneous equilibrium of total demand and total supply both on goods and capital markets:

\[
Y_{IS} = A + m_G \cdot G_a + m_T \cdot T_a + m_{TR} \cdot TR_a - \beta \cdot m_f \cdot r
\]

\(^1\) An autonomous value – that part of some revenues or expenditures, that does not depend on the level of income, but depends on other variables, other than income.
\[ A = \frac{C_a}{1 - (c_y \cdot (1 - t) + \eta)} = m_a \cdot C_a \] – the multiplied value of autonomous consumption, which are difficult to govern;

\[ m_G = \frac{1}{1 - (c_y \cdot (1 - t) + \eta)} \] – the multiplier of autonomous government purchases;

\[ m_f = \frac{-c_y}{1 - (c_y \cdot (1 - t) + \eta)} \] – the multiplier of autonomous taxes;

\[ m_{TR} = \frac{c_y}{1 - (c_y \cdot (1 - t) + \eta)} \] – the multiplier of autonomous transfers;

\[ m_I = \frac{1}{1 - (c_y \cdot (1 - t) + \eta)} \] – the multiplier of autonomous investment.

In the IS function, one variable, namely interest rate \((r)\), is endogenous, and three other variables, government purchases \((G_a)\), taxes \((T_a)\) and transfers \((TR_a)\), are exogenous. The IS curve demonstrates the dependency between \(Y\) and \(r\) under condition that in current period all three parameters of the fiscal policy are set. Their changes cause a shift of the IS curve to right (when expenditures increase) or to the left (when expenditures increase).

Additionally, the table 2.1 demonstrates the multiplier effects in an open economy.

**Table 2.1**

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<th>Multiplier of...</th>
<th>Formula for calculation</th>
<th>Total expenditures influence</th>
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<td>autonomous expenditures</td>
<td>( m_a = \frac{1}{1 - (c_y \cdot (1 - t) - \mu + \eta)} )</td>
<td>( \Delta Y_D = m_a \times \Delta A_a )</td>
</tr>
<tr>
<td>investment</td>
<td>( m_I = \frac{1}{1 - (c_y \cdot (1 - t) - \mu + \eta)} )</td>
<td>( \Delta Y_D = m_I \times \Delta I_a )</td>
</tr>
<tr>
<td>government purchases</td>
<td>( m_G = \frac{1}{1 - (c_y \cdot (1 - t) - \mu + \eta)} )</td>
<td>( \Delta Y_D = m_G \times \Delta G_a )</td>
</tr>
<tr>
<td>transfers</td>
<td>( m_{TR} = \frac{c_y}{1 - (c_y \cdot (1 - t) - \mu + \eta)} )</td>
<td>( \Delta Y_D = m_{TR} \times \Delta TR_a )</td>
</tr>
<tr>
<td>autonomous taxes</td>
<td>( m_f = \frac{-c_y}{1 - (c_y \cdot (1 - t) - \mu + \eta)} )</td>
<td>( \Delta Y_D = m_f \times \Delta T_a )</td>
</tr>
</tbody>
</table>
| balanced budget | \( m_{BD} = \frac{1 - c_y}{1 - (c_y \cdot (1 - t) - \mu + \eta)} \) | \( \Delta Y_D = m_{BD} \times \Delta G_a \) (on conditions that: \(\Delta T = t_2 \cdot Y_2 - t_1 \cdot Y_1\), \(\Delta G_a = \Delta T_a\))

(when extra government purchases are financed by additional autonomous tax)

\[ m_{BD} = \frac{1 - c_y}{1 - (c_y - \mu + \eta)} \] (when extra government purchases are financed by additional income tax)
Note: MPC – marginal propensity to consume as to disposable income; \( t \) – income tax rate; \( MPC \times (1-t) \) – marginal propensity to consume as to gross income; \( \mu \) – marginal propensity to consume import goods and services; \( \eta \) – marginal propensity to induced investment.

- **The Haavelmo theorem** affirms that an increase in the public expenditures, which is financed fully over additional income taxes, primarily results in the same product increase, thus \( \Delta G = \Delta T = \Delta Y \).

**The Haavelmo theorem** is true under the condition: \( \mu = \eta = 0 \).

2. **The Haavelmo theorem on money market. Deriving the LM curve**

The liquidity preference function: \( L(Y,r) = L_a + l_y \cdot Y - l_r \cdot r \), where \( l_y \) – the coefficient showing, what part of their income the businesses prefer to keep in liquid form, intending for current purchases of goods and services (this coefficient is opposite value to the money velocity); \( l_r \) – the sensitivity of liquidity preference (namely assets money demand) to the real interest rate.

The real money supply is \( M_s / P \), where the nominal money supply \( M_s \) is exogenously established in the framework of monetary policy.

The money market equilibrium: \( M_s / P = L(Y,r) \).

The LM function demonstrates the balance of money demand and money supply:

\[
Y_{LM} = -L + \frac{l_r}{l_y} \cdot r + \frac{1}{l_y} \cdot \left( \frac{M_s}{P} \right),
\]

where \( L = \frac{L_a}{l_y} \), coefficients \( \frac{l_r}{l_y} \) and \( \frac{1}{l_y} \) are constant.

3. **Co-equilibrium of three markets in the IS-LM model and its changes**

Co-equilibrium of the markets of goods, capital and money in closed economy is presented on the figure 2.1. The equilibrium parameters are: \( r_c \) – the equilibrium real interest rate, \( Y_{Re} \) – the equilibrium real income.

Now examine the consequences of active regulative policy in short-run.

1. **The impact of fiscal policy on equilibrium in the IS-LM model.** Under the expansionary (simulative) fiscal policy the IS curve shifts right by \( m_A \cdot \Delta A_a \) (where \( m_A \) – the multiplier of autonomous expenditures, \( \Delta A_a \) – change in the value of the relevant part of autonomous expenditures. The result depends on the segment, where the IS curve intersects with LM curve: horizontal, slopped or vertical.

*The horizontal segment of the LM curve* characterizes the statement of «the liquidity trap» in an economy. The expansionary fiscal policy results in increase of the real income \( Y_{Re} \) when real interest rate \( r_c \) stays the same. On this segment fiscal policy is absolutely effective and crowding-out effect equals zero.

- **Liquidity trap** – the statement of absolutely elastic liquidity preference with re-
speak to real interest rate. It is characterized with the lowest level of interest rate, which undermines incentives to invest in the assets alternative to money, like bonds and deposits. Under the liquidity trap, additional money supply automatically transforms into extra assets money demand:

*The slopped segment of the LM curve* implies the transitional state of the economy, characterized by incomplete employment of resources. On this segment the fiscal expansion causes the increase in real income $Y_{Re}$ and real interest rate $r_e$. Interest rate rise causes a decrease in investment. The positive effect of fiscal expansion is partially offset by the so-called "crowding out effect". On this segment, the fiscal policy is relatively effective. Its effectiveness is directly dependent on the value of the multiplier of autonomous expenditures $m_A$, and the sensitivity of money demand to the interest rate $l_r$, and inversely proportional to the sensitivity of investment to the interest rate $\beta$.

- **Crowding-out effect** — the offset in aggregate demand resulting from the expansionary fiscal policy. Increased public purchases or tax reduction leads to an increase in the demand for money and the rise in real interest rate in the economy, which negative affects private domestic investment.

*The vertical segment of the LM curve* characterizes the statement of full employment in an economy. The fiscal expansion results in increase in real interest rate $r_e$ without changes in real income $Y_{Re}$. In this case, additional public expenditure crowds out private investment by the same amount, crowding-out effect equals 100%. In this statement, fiscal policy is completely ineffective.

2. **The impact of monetary policy on equilibrium in the IS-LM model.** Under expansionary (stimulating) monetary policy LM curve shifts to the right. The result depends on what part of the LM curve intersects the IS curve. *The negatively slopped segment of the IS curve* characterizes the statement of economy where the private investment is sensitive to the real interest rate. On this segment, monetary expansion causes an increase in real income $Y_{Re}$ and a decrease in the real interest rate $r_e$. The effectiveness of monetary policy is directly dependent on the sensitivity of investment to the real interest rate $-\beta$, and inversely dependent on the sensitivity of money demand to the interest rate $-l_r$.

*The vertical segment of the IS curve* characterizes the statement of «the investment trap» in an economy. On this segment, the monetary expansion results in reduction in the real interest rate $r_e$, which however does not affect the increase in investment and real income $Y_{Re}$. In this statement, monetary policy is completely ineffective.

- **Investment trap** — the statement of absolutely inelastic investment with respect to real interest rate. In this case the size of the investment is affected by other factors associated with institutional (rules, norms) or macroeconomic (demand for goods and services) environment.

Based on the IS-LM model we can derive the aggregate demand curve, AD (figure 2.2). The increase in the general price level from $P_1$ up to $P_2$, leads to a decrease in the real money supply bringing the LM curve shifts to the left. Equilibrium moves from the point $E_1$ to the point $E_2$, and equilibrium level of the real income decreases from $Y_1$ to $Y_2$. Thus, the higher price level ceteris paribus corresponds to the lower level of real expenditures.

- **Aggregate demand** — different quantities of domestic goods and services that aggregate sectors of households, firms, the government and foreign sector can and wish to buy at each price level.

The negative slope of the AD curve is attributable to three effects:
• **The Real Interest Rate Effect (by Keynes)** – when the price level in the country increases, the demand for money grows, and under a constant supply of money it leads to an increase in the real interest rate. This in turn has a negative effect on investment, which is an element of real aggregate demand.

• **The Wealth Effect** (Pigou's) – when the price level in the country increases, the real value of the nominal assets in wealth (cash, bank deposits, bonds) reduces. According to the F. Modigliani's life cycle hypothesis, the consumption expenditures depend on the accumulated wealth and expected income throughout life. Therefore, a decrease in real wealth reduces real consumption.

• **The Real Exchange Rate Effect** (by R. Mundell and M. Fleming) – when the price level in the country increases (under the condition that the prices of foreign substitutes and exchange rate remain unchangeable), domestic products become less attractive to buyers both on domestic and foreign markets. The demand shifts towards foreign goods. As a result, the imports in given country increase and exports decline. Net exports, which is an element of aggregate demand, reduces.

### 2.3.2. Deriving the aggregate supply function for the goods market (AS) based on the short-run labor market equilibrium

Aggregate supply curve AS may be derived on the basis of three equations concerning the labor market and its relationship with the goods and services market, namely: 1) the A. Phillips equation; 2) the dependency between the price level and the wage rate; 3) the A. Okun' equation.

1. **The A. Phillips equation** represents the relationship between the wage rate and employment level:

   \[ W = W^* \left( 1 + \alpha \frac{N-N^*}{N^*} \right), \]

   where \( W \) – the actual wage rate, \( W^* \) – the wage rate under full employment, \( N \) – the number of actually employed persons, \( N^* \) – the potential number of employed, \( \alpha \) – coefficient characterizing the wages sensitivity to labor market conditions (excessive or under-employment).

   Taking into account the relationship of the number of employees and the unemployment level, \( \frac{N^*-N}{N^*} = u - u_n \), we can modify the Phillips equation:

   \[ W = W^* \left( 1 - \alpha \cdot (u - u_n) \right). \]

2. Using the pricing method "cost of labor plus markup", we can establish the dependency between the price level and the wage rate:
\[ P = (1 + \lambda) \cdot \frac{N \cdot W}{Y} = (1 + \lambda) \cdot \tau \cdot W, \]

where \( \lambda \) – price markup to wage costs, \( \tau = \frac{N}{Y} \) – units of labor spent on average per unit of national income.

By substitution \( W \) in this equation with the modified equation Phillips, we get:

\[ P = (1 + \lambda) \cdot \tau \cdot W \ast \left(1 - \alpha \cdot (u - u_n)\right). \]

The price level at full employment is: \( P^\ast = (1 + \lambda) \cdot \tau \cdot W \ast \). The ratio of the actual price level to its level under full employment:

\[ \frac{P}{P^\ast} = 1 - \alpha \cdot (u - u_n). \]

3. The A. Okun equation is an empirically proved (“rule of thumb”) negative relationship between cyclical unemployment and relative output gap:

\[ \frac{Y - Y_F}{Y_F} = -\gamma \times (u - u_n), \]

where \( Y \) – actual level of income (output, real GDP); \( Y_F \) – potential level of income; \( u \) – actual rate of unemployment, \( u_n \) – natural rate of unemployment, \( u - u_n \) – cyclical rate of unemployment, \( \gamma \) – Okun's coefficient, demonstrating the relative losses in GDP compared to its potential level when the actual rate of unemployment exceeds its natural rate by 1%. For the United States since 1955 this coefficient was estimated to be the range 2 to 3.

4. The aggregate supply equation is derived based on above three equations and shows the relationship between the actual GDP and the price level. The more the actual GDP, the higher the employment rate, lower unemployment, higher wage rate and the higher the level of prices in the country. By deriving \( u - u_n \) from the Okun equation (p. 3) and substituting it into the final equation of p. 2, we obtain the equation of short-term aggregate supply in static form:

\[ \frac{P}{P^\ast} = 1 + \omega \cdot (Y - Y_F), \]

where \( \omega = \frac{\alpha}{\gamma \times Y_F} \) – coefficient reflecting the price response to «output gap Assume that the initial price level equals one, \( P^\ast = 1 \), and the aggregate supply equation becomes even more simple:

\[ P = 1 + \omega \cdot (Y - Y_F). \]

Now we can determine the aggregate supply on goods market as a dependency between output and price level.

- **Aggregate supply** – different quantities of goods and services that aggregate sectors of firms can and wish to produce and sell at each price level.

### 2.4. Problems

**Problem 1.** Examine the following case. Before the economic crisis of 2008-2009, Russia was an economy with two surpluses: the trade account surplus (\( NX > 0 \)) and the state budget surplus (\( BS = -BD \)). Favorable conjuncture for petroleum and other energy resources in world markets resulted in inflows of sustainable export revenues into Russian economy. Russian banks and non-financial corporations borrowed actively abroad. They expected ruble exchange rate would further increase. Foreign currency inflows to national economy have enlarged considerably. The Central Bank of Russia purchased exchange currency to restrain ruble revaluation and avoid worsening of national competitiveness, actually accomplishing a massive ruble intervention. The Russian gov-
ernment formed Stabilization funds in foreign currency deposits and advanced countries’ public bonds to sterilize excessive money supply.

Describe Russian economics conditions in terms of main economic identities presented above. What were the conditions in advanced countries at that time?

**Problem 2.** Suppose some economy has shown the following annual results: gross income Y=2800; consumption: C=2000; government purchases G=600; transfers from government to public TR=300; interests paid for public bonds at the disposal of private holders N=100; taxes T=800; net export NX=−300.

Determine the value of: a) gross internal private investment; b) deficit/surplus of state budget; c) income at the disposal of private sector; d) private savings.

*Answer:* a) \( I_g = 500 \); b) \( BD = 200 \); c) \( Y_d = 2400 \); d) \( S_p = 400 \).

**Problem 3.** Suppose some economy has shown the following annual results: C=2000; G=300; TR=240; T=450; trade balance surplus is equal to 180. Gross income is equal to 3100.

Evaluate the share of gross private savings aimed at financing gross internal private investment.

*Answer.* 0.7 (or 70%).

**Problem 4.** Suppose some economy has shown the following annual results: consumption C=2400; gross internal investment I=700; government purchases G=800; social transfers TR=100; paid interest for public debt bonds at the disposal of private sector N=200; autonomous taxes T=800. Budget deficit is planned to be covered for 80% by bonds offering, and for 20% by money emission.

What share of new private savings should be presented in the form of public bonds purchases?

*Answer.* 24%.

**Problem 5.** What economic conditions are not taken into account at the Haavelmo theorem?

**Problem 6.** Suppose the economy of some country is characterized by the following data:

\[
Y = C + I + G + NX, \\
C = 200 + 0.6 \times Y_d, \\
I = 400 - 2000 \times r, \\
NX = 100 - 0.1 \times Y, \\
M_d / P = 0.5 \times Y - 3000 \times r.
\]

Tasks:

a) Derive the equations for IS and LM functions;

b) Let \( T = 400 \), \( G = 300 \), \( M_s = 600 \), and \( P = 1 \). Evaluate the equilibrium income and the equilibrium interest rate for these conditions;

c) Develop the equation for AD curve as the relation between the real expenditures and the real money supply, autonomous taxes and government purchases;

d) Suppose the Government has decided to increase aggregate demand in short-run by 180 units. How much should it change the autonomous taxes or the public purchases of goods and services to achieve this aim? Estimate the crowding-out effect in this case. Determine equilibrium interest rate change.

e) Suppose not the Government but the Central Bank has set the goal to increase the income by 180 units by means of monetary policy. How much it has to change money supply in the short-run period? Estimate the changes in equilibrium level of the interest rate and investment in this case.

*Answer.* a) IS: \( Y_{IS} = 1400 - 1.2 \times T + 2 \times G - 4000 \times r \); LM: \( Y_{LM} = 2 \times \frac{M_s}{P} + 6000 \times r \); b) \( Y_E = 1392 \) units; \( r_E = 0.032 \) (or 3.2%); c) AD: \( Y_{AD} = 840 - 0.72 \times T + 1.2 \times G + 0.8 \times \frac{M_s}{P} \); d) \( \Delta T = -250 \) units, or \( \Delta G = +150 \) units. The crowding-out effect is equal to 120 units in both cases. \( r_E = 0.062 \) (or 6.2%); e) \( \Delta M_s = +225 \) units, \( \Delta r = -4.5% \); \( \Delta I = +90 \) units.
Problem 7. Imagine some closed economy with the following characteristics: consumption function is \( C = 600 + 0.6 \times (Y - T) \); investment function is \( I = 500 - 1600 \times r \) \((r\) is expressed as a fraction); tax function is \( T = 100 + 0.25 \times Y \); government purchases function is \( G = 400 + 0.15 \times Y \). Demand for real money is \( (M/P)_d = 0.5 \times Y - 3000 \times r \); money supply is \( M_s = 1600 \); price level is \( P = 2 \).

Suppose the government has increased autonomous government expenditures by 200 units. Estimate the crowding-out effect. What should the Central Bank undertake to neutralize this effect entirely?

*Answer.* The crowding-out effect is equal to 200 units. The Central Bank should increase money supply by \( \Delta M_s = 500 \) units.
UNIT 3. Macroeconomic equilibrium in an open economy

3.1. Fundamentals of an Open Economy

Balance of Payments equilibrium under flexible and fixed exchange rate. Consequences of monetary and fiscal policy in IS-LM-BP model

- **Balance of Payments** – a table reflecting the results of the trade and financial transactions of given country with other countries, leading to cash flows from this country abroad and from other countries to this country.

  Balance of payments consists of two accounts: 1) the current account; 2) the capital account or financial account.

  The main component of the current account is the country’s trade balance (net flows of money as a result of the country's foreign trade with other countries). Two other components of it are the factor incomes (earnings on foreign investments minus payments made to foreign investors) and net cash transfers.

  The capital account reflects net change in foreign ownership of national assets. A surplus in the capital account means outflows of money by means of net lending to other countries or increase in ownership of foreign assets.

  Macroeconomics examines rough goods presentation of balance of payments, which includes two important components: the trade balance \( NX \) and capital account (which is the difference between the investment and the budget deficit and savings: \( I + BD - S \)). In sum, they are equal to the change in foreign exchange reserves \( \Delta R \):

  \[
  NX + (I + BD - S) = \Delta R.
  \]

  When the change in reserves is zero: \( NX + (I + BD - S) = 0 \).

  Balance of payments as well can be presented as equality of net goods exports and net capital exports, \( NX = S - (I + BD) \).

  ✓ **Nominal exchange rate** - the amount of foreign currency that can be obtained for 1 unit of the national currency in exchange.

  ✓ **Real exchange rate** - the number of foreign goods that can be purchased instead of 1 unit of domestic goods under the current price level in the country and abroad, and actual nominal exchange rate. When we calculate the relative price of representative market basket in two countries, the real exchange rate is called purchasing power parity (PPP).

  The real exchange rate \( \varepsilon_r \) can be calculated using the formula:

  \[
  \varepsilon_r = \varepsilon_n \cdot \frac{P}{P^*},
  \]

  where \( \varepsilon_n \) – nominal exchange rate, \( P \) – general price level in the domestic country, \( P^* \) – price level abroad.

  "Terms of trade" – is an inverse indicator to the real exchange rate:

  \[
  \theta = \frac{\text{The average price of foreign goods}}{\text{the average price of domestic goods}} = \frac{P^*}{\varepsilon_n \cdot P}.
  \]

  If \( \theta > 1 \), foreign substitutes are more expensive than domestic goods when their prices converted into foreign currency on the basis of the nominal exchange rate. This means that the demand for domestic goods will increase and the demand for foreign goods will decline in both domestic and foreign markets. When \( \theta > 1 \) terms of foreign trade are favorable for national country. In such
circumstances net exports (exports minus imports) will increase, that will cause net inflows of foreign currency and decline in its exchange rate. The national currency exchange rate will rise, and \( \theta \) will decline, tending to 1.

If \( \theta < 1 \), foreign substitutes are less expensive than domestic goods. The terms of foreign trade are unfavorable for national country. Net exports will decline, outflows of foreign currency will take place, and foreign currency exchange rate will increase while reducing national currency exchange rate. \( \theta \) will grow, tending to 1.

- **“The law of one price”:** when institutional constraints are absent and there exists the perfect cross-border mobility of resources and goods, uniform prices for all tradable goods and services are installed on the domestic and international markets. Under these conditions real exchange rate equals to nominal exchange rate.

  Limitations for the law of one price: 1) imperfect mobility of goods and resources; 2) tradable and non-tradable goods.

### 3.2. Model of balance of payments in a small open economy with perfect capital mobility

In such an economy, due to the perfect mobility of capital, the real interest rate is set at the level of the world rate: \( r = r^* \). National currency market equilibrium takes the form:

\[
S - I(r^*) - BD = NX(\varepsilon_r),
\]

where investment (I) is an inversely related to the real interest rate (r), and net exports (NX) is inversely related to the real exchange rate (\( \varepsilon_r \)). Equilibrium on national currency market is established due to adjustment of real exchange rate (\( \varepsilon_r^* \)).

In this model **expansionary fiscal policy** leads to an increase in the budget deficit and shifts the \( S - I(r^*) - BD \) curve to the left. The domestic currency supply is reducing, and the exchange rate of the national currency is rising. This causes a reduction in net exports - \( NX(\varepsilon_r) \). Thus additional government spending crowd out the same amount of net exports.

**Expansionary monetary policy** doesn’t affect the equilibrium in this model, because fall of the internal interest rate leads to an outflow of money to other countries. Capital rushes into other countries in search of the “interest arbitrage”, and its perfect mobility contributes to this. As a result, emitted money is distributed evenly between countries. So as domestic country is relatively small in the large competitive environment, its emission policy is unlikely to have a significant impact on the global interest rate and, accordingly, does not affect the level of domestic investment and the domestic currency supply in the currency market.
3.3. The R. Mundell – M. Fleming model for a small open economy (IS-LM-BP model)

This model is an extended version of the IS-LM model. In this model, balance of payments equilibrium is added to the equilibrium of commodity and money markets, and the BP curve represents it.

The BP curve brings together different combinations of real income and real interest rate under which the net exports of goods and services equals to net capital outflow in given country: \( NX = NEK \). Exports of goods and services is directly related to the real income in foreign countries and inversely related to the real exchange rate of the national currency: \( X(Y^*, e^-) \). Imports of goods and services is directly related to the domestic real income and to the real exchange rate: \( Z(Y^r, e^r) \). Thus, the function of net exports, which is the difference between exports and imports \( (NX = X - Z) \), takes the form: \( NX(Y^r, e^- r^*) \).

Net capital outflow responds to the difference in interest rates in different countries and takes into account the expected changes in the exchange rate: \( r^* - (r + e^-) \). The higher the real exchange rate the less its expected changes are. So net export of capital is directly related to the foreign real interest rate \( (r^*) \) and to the real exchange rate \( (e^-) \) and it is inversely related to the domestic interest rate \( (r) \): \( NEK(r, r^*, e^r) \).

When we take into account only the income and the interest rate in the domestic country, the balance of payments takes the form: \( NX(Y^-) = NEK(r^-) \). And the BP curve has a positive slope (Figure 3.2).

![Figure 3.2. Equilibrium in balance of payments, deriving the BP curve](image)

Properties of the curve BP:
The slope of the BP curve is inversely related to the degree of capital mobility: the higher the mobility of capital, the flatter the BP curve. With perfect capital mobility, the curve becomes horizontal.

The curve shifts to the right–down when the income in other countries increases and vice versa. The curve shifts to the left–up when the exchange rate or the world interest rate increases and vice versa.

3.4. The demand managing in a small open economy with perfect mobility of capital: short-term balance change in the IS-LM-BP model

Assumptions of the model: 1) perfect capital mobility; 2) general price level rigidity (short term Keynesian equilibrium); 3) the deviation of the economy from the state of full employment calls the management of aggregate demand; 4) the effects of monetary and fiscal policy depend on the exchange rate regime.

1. Expansive fiscal policy under a floating exchange rate (Figure 3.3):
   - As a result of increase in government expenditures or decrease in taxes the IS curve shifts to the right (IS₁ → IS₂);
   - an increase in the demand for money, when the money supply is constant, will cause the rise of the internal interest rate from r* to r', and income will grow from Yᵣ₁ to Yᵣ₂;
   - while the domestic interest rate is higher the foreign interest rate (r' > r*), the capital will flow into the domestic country from abroad;
   - capital inflows will result in growth of foreign currency supply, and foreign currency will depreciate, while national currency appreciate (ε ↑);
   - an increase in the exchange rate will lead to a deterioration of “the terms of trade” and negative impact on the net exports (NX ↓). Ultimately the IS curve shifts to the former position IS₁. Income returns to the former level Yᵣ₁.

   Thus, in an economy with perfect capital mobility and a floating exchange rate regime, fiscal policy is ineffective.

2. Expansive monetary policy under a floating exchange rate (Figure 3.4):
   - As a result of increase in money supply the LM curve shifts to the right (LM₁ → LM₂);
   - an increase in the money supply, when the demand for money is constant, will cause lowering the internal interest rate from r* to r', and income will grow from Yᵣ₁ to Yᵣ₂ due to an increase in domestic investment;
while the domestic interest rate is below the foreign interest rate \((r' < r^*)\), the capital will flow out of the country abroad;

capital outflows will result in declining the foreign currency supply, and foreign currency will appreciate, while national currency depreciates \((\varepsilon \downarrow)\). The central bank in a floating exchange rate regime doesn’t prevent the establishment of the new equilibrium exchange rate;

reducing the exchange rate will lead to an improvement in “the terms of trade”, which will result in ascending the net exports \((NX \uparrow)\). This causes a shift of the IS curve to the right \((IS_1 \rightarrow IS_2)\). As a result, the interest rate returns to the foreign level – \(r^*\), and income grows up to \(Y_{r_2}\).

Thus, in an economy with perfect capital mobility and a floating exchange rate regime, monetary policy is highly effective.

3. Expansive fiscal policy under a fixed exchange rate (Figure 3.5):

An increase in government expenditures or decrease in taxes shifts the IS curve to the right \((IS_1 \rightarrow IS_2)\);

the domestic interest rate grows from \(r^*\) to \(r'\), and the income grows up to \(Y_{r'}\);

foreign capital rushes to the domestic economy in search of interest arbitrage \((r' > r^*)\);

foreign currency inflow causes a rise in the exchange rate \((\varepsilon \uparrow)\);

since the central bank pursues a regime of fixed exchange rate, it will buy foreign currency in the foreign exchange market impeding the national currency appreciation. Such a policy leads to increase in the money supply within the country;

the LM curve shifts to the right \((LM_1 \rightarrow LM_2)\). The interest rate returns to its previous level. The income grows up to \(Y_{r_2}\).

Thus, in an economy with perfect capital mobility and a floating exchange rate regime, fiscal policy is highly effective.

4. Expansive monetary policy under a fixed exchange rate (Figure 3.6):

The money supply increase shifts the LM curve shifts to the right \((LM_1 \rightarrow LM_2)\);

lowering the internal interest rate from \(r^*\) to \(r'\) causes the domestic investment rise and income enlargement to \(Y_{r'}\);

the internal interest rate reduction compared to the world interest rate \((r' < r^*)\) induce the capital to flee the country;

demand for foreign currency grows and it appreciates while national currency depreciates \((\varepsilon \downarrow)\). For stabilizing the situation the central bank under the fixed exchange regime will sell foreign currency, withdrawing money from circulation;

reduction of the national money supply shifts the LM curve to the left to its former state

---

Figure 3.5. The consequences of the fiscal shock under a fixed exchange rate

Figure 3.6. The consequences of the monetary shock under a fixed exchange rate
(\(LM_2 \rightarrow LM_1\)), the interest rate and income return to their previous level. The exchange rate remains unchanged.

Thus, in an economy with perfect capital mobility and a fixed exchange rate regime, monetary policy is ineffective.

### 3.5. Problems

**Problem 2.1.** In some small country with complete mobility of capital the function of consumer demand for domestic goods is given by: \(C = 100 + 0.7 \cdot (Y - T)\), and the function of demand for imported goods: \(Z = 300 + 0.3 \cdot Y + 2 \cdot \varepsilon\) (where \(\varepsilon\) – real exchange rate). The investment function: \(I = 600 - 40 \cdot r^*\) (where \(r^*\) – the world real interest rate), the function of exports:

\[X = 500 + 0.1 \cdot Y - 3 \cdot \varepsilon\].

Autonomous taxes are 100, and the income tax rate is 20%. The government adheres to the policy of a balanced budget. The real interest rate on world capital markets is 5%.

Tasks: A) Let the potential income in given country equals 1000 in real terms. Determine the equilibrium real exchange rate, the state of the current account and the capital account of the balance of payments in long run. B) Let the government took the course of expansionary fiscal policy and increased government purchases by 50 units. How will the equilibrium real exchange rate and the state of balance of payments accounts change? C) Let the government instead of p."b" has imposed imports quotas, that resulted in decrease in the value of imports by 50 units. How will the equilibrium real exchange rate and the state of balance of payments change? All answers provide graphic illustrations.

**Answer:** A) \(\varepsilon = 58\), current account: \(NX = -290\) units, capital account: \(NK = 290\) units; b) \(\varepsilon = 68\), current account: \(NX = -340\) units, capital account: \(NK = 340\) units; c) \(\varepsilon = 68\), current account: \(NX = -290\) units, capital account: \(NK = 290\) units.

**Problem 2.2. Mundell-Fleming model with perfect capital mobility.** In some small country with perfect capital mobility the function of consumer demand for domestic goods is given by: \(C = 120 + 0.76 \cdot (Y - T)\), and the function of demand for imported goods: \(Z = 0.17 \cdot Y\). The investment demand of domestic entrepreneurs is given by: \(I = 200 - 7.2 \cdot r\) (where \(r\) – domestic real interest rate). The government purchases of goods and services are 360 units. The budget revenues are formed by a 25% income tax. Goods exports are 220 units. Money demand for transactions is 25% of income, assets demand for money is given as function: \(L_{sp} = 60 - 2 \cdot r\). Real money supply is 400 units. The real interest rate on world capital markets is \(r^* = 6\%\).

Tasks: A) Derive equations of the \(IS\), \(LM\) and \(BP\) curves, construct them on chart. Determine the equilibrium level of income, the domestic interest rates, the state of government budget and the state of trade balance; B) What changes will occur in the economy under floating and fixed exchange rates? What equilibrium parameters will be established?

**Answer:** A) \(Y_{IS} = 1500 - 12 \cdot r\); \(Y_{LM} = 1360 + 8 \cdot r\); BP: \(r^* = 6\%\). \(Y_E = 1416\) units; \(r = 7\%\).

\(BD = -6\) units, \(NX = -20,72\) units; b) because of capital inflows in economy exchange rate will rise. Under floating exchange rate net exports will decrease by 12 units, the IS curve will shift to the left by 20 units (\(\Delta Y_{IS} = m_{nx} \cdot \Delta NX = 1.6\cdot 12\)), the equilibrium income will decrease by 8 units and will amount to 1408 units, the interest rate will reach the world level (6%). Under a fixed exchange rate the central bank will increase the money supply by 5 units by restraining the growth of the national currency exchange rate. The LM curve will shift to the right by 20 units, the equilibrium income will increase by 12 units and will reach 1428 units, the domestic interest rate will be equal to the world interest rate (6%).
UNIT 4. Economic growth and its modeling

4.1. The R. Solow-T. Swan Growth Model

This model was developed by Robert Solow and T.W. Swan in 1956.

1. It is based on following production function (PF):

\[ Y(t) = F(K(t), A(t) \times L(t)) \]  \hspace{1cm} (1)

where \( Y \) – output, \( K \) – capital, \( L \) – labor, \( A \) – knowledge or the «effectiveness of labor», \( t \) – time. \( A(t) \times L(t) \) is so called «effective labor», and the technical progress here is «labor augmenting» or Harrod-neutral.

2. There are some assumptions concerning production function:
   - homogeneous character;
   - constant returns to scale:

\[ \forall c \geq 0, \ F(cK, cAL) = cF(K, AL). \]  \hspace{1cm} (2)

It means that: a) the advantages of specialization are exhausted, and the economy is sufficiently big; b) other factors, such as natural resources and land, have no impact on output.

By dividing both parts of the equation (1) by \( AL \), we get he expression:

\[ \frac{Y}{AL} = \frac{1}{AL} F(K, AL) = F \left( \frac{K}{AL} \right), \]  \hspace{1cm} (3)

that can be interpreted as follows: output per unit of effective labor \( (y = \frac{Y}{AL}) \) is a function of capital per unit of effective labor \( (k = \frac{K}{AL}) \). And the equation (3) takes the intensive form:

\[ y = f(k). \]  \hspace{1cm} (4)

- **declining but positive returns to capital** (and to «capital per unit of effective labor») as capital rises:

\[ f(0) = 0, \ MP_{k} = f'(k) > 0, \ f'^{*}(k) < 0, \]  \hspace{1cm} (5)

where \( MP_{k} = \frac{\partial Y}{\partial K} = \frac{\partial y \cdot AL}{\partial k \cdot AL} = \frac{\partial y}{\partial k} \) is the marginal product of capital.

- **production function satisfies the Inada conditions**: \( \lim_{k \to 0} f'(k) = \infty \), \( \lim_{k \to \infty} f'(k) = 0 \).

3. Cobb-Douglas function might be considered as an appropriate case of PF:

\[ F(K, AL) = K^{\alpha}(AL)^{1-\alpha}, \]  \hspace{1cm} (6)

In intensive form:

\[ f(k) = k^{\alpha}. \]  \hspace{1cm} (7)

Marginal product of capital: \( MP_{k} = f'(k) = \alpha k^{\alpha-1} = \alpha \times f(k) \frac{a-1}{\alpha} \).  \hspace{1cm} (8)

All the above-listed requirements are satisfied.

4. Dynamics of the model with constant inputs: \( \dot{k}(t) = s \times f(k(t)) - \delta \times k(t) \),  \hspace{1cm} (9)

where \( \dot{k}(t) \) – time rate change of the capital stock per unit of effective labor; \( s \) – saving rate, i.e. average part of output that householders and firms intend to propose as a source for investment; \( \delta \) – depreciation rate, the part of capital that wears out; both parameters are exogenous and constant.

**Equilibrium in the model**: \( s \times f(k^*) = \delta \times k^* \),  \hspace{1cm} (10)

\( k^* \) is steady level of \( k \), under which actual investment is equal to break-even investment.

Parameters of equilibrium for Cobb-Douglas function: \( k^* \frac{1}{\frac{a}{\delta}}; \ y^* \frac{1}{\frac{a}{\delta}} \).

5. Factors of economic growth:
• growth in labor: \( \dot{L} = n \times L(t) \), \( n = \frac{\dot{L}(t)}{L} = \frac{d \ln L(t)}{dt} \);

• growth in knowledge: \( \dot{A} = g \times A(t) \), \( g = \frac{\dot{A}(t)}{A} = \frac{d \ln A(t)}{dt} \).

Parameters \( n \) and \( g \) are considered exogenous.

**Dynamics of the model with growing inputs:**

\[
\dot{k}(t) = s \times f(k(t)) - (n + g + \delta) \times k(t)
\]

**Equilibrium in the model:**

\[
s \times f(k^*) = (n + g + \delta) \times k^*
\]

Parameters of equilibrium for Cobb-Douglas function:

\[
k^* = \left( \frac{s}{n + g + \delta} \right)^{\frac{1}{1-\alpha}}; \quad y^* = \left( \frac{s}{n + g + \delta} \right)^{\frac{\alpha}{1-\alpha}}.
\]

6. **Balanced growth path** – a situation of constant rate growth of each variable in the model (Table 4.1).

<table>
<thead>
<tr>
<th>Variables</th>
<th>Rate of Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital per unit of effective labor ( k = K/AL )</td>
<td>0</td>
</tr>
<tr>
<td>Capital per worker ( k = K/L )</td>
<td>( g )</td>
</tr>
<tr>
<td>Stock of capital ( K = k \times (AL) )</td>
<td>( n + g )</td>
</tr>
<tr>
<td>Output per unit of effective labor ( y = f(k) = Y/AL )</td>
<td>0</td>
</tr>
<tr>
<td>Output per worker ( Y/L = y \times A )</td>
<td>( g )</td>
</tr>
<tr>
<td>Stock of effective labor ( AL )</td>
<td>( n + g )</td>
</tr>
<tr>
<td>Output ( Y = y \times (AL) )</td>
<td>( n + g )</td>
</tr>
</tbody>
</table>

7. **Shifts in equilibrium under the impact of \( s \) and \( \delta \) changes** are presented in Figure 4.1.

![Figure 4.1. The Solow growth model: equilibrium and its change](image-url)
8. Golden rule level of capital stock by E. Phelps affirms that a steady level of capital per unit for effective labor should ensure maximum consumption.

Consumption per unit of effective labor: \( c(k) = (1-s) \times f(k) = f(k) - (n + g + \delta) \times k \rightarrow \text{max.} \) \( c'(k) = 0, \ c''(k) < 0. \)

\[
f'(k^{**}) = n + g + \delta, \quad (11)
\]

or the same: \( MP_k = n + g + \delta \) \( (12) \)

It is represented on the figure 4.2:

For: \( k^* = k^{**} \):

\[
\begin{align*}
& s \times f(k) = (n + g + \delta) \times k; \\
& f'(k) = n + g + \delta. \\
\Rightarrow & \quad s = \frac{f'(k) \times x}{f(k)}.
\end{align*}
\]

Golden rule: optimal saving rate is equal to elasticity of output with respect to capital. For Cobb-Douglas function: \( s = \alpha \).

When \( k^* < k^{**} \) (the steady level is less than the golden level), the state should conduct an economic policy aimed at increasing the saving rate (\( s \uparrow \)).

When \( k^* > k^{**} \) (the steady level is more than the golden level), the state should conduct an economic policy aimed at decreasing the saving rate (\( s \downarrow \)).

9. Convergence («catch-up effect») – tendency of a more rapid economic growth in developing (poor) countries than in developed (rich) countries because of the smaller initial rate of return on capital in the former ones and their tendency to get a balanced growth path. Moreover, poor countries can replicate available technologies from rich countries thus gaining time. As a result, all economies will eventually converge in terms of per capita income.

10. Speed of convergence to the steady level: \( \lambda = \left[ 1 - \alpha_k(k^*) \right] (n + g + \delta) \), \( (13) \) where \( \lambda \) is an annual rate of approaching \( k \) to \( k^* \). For reducing the distance by half, the time \( t^* = \ln(0.5) / \lambda \approx 0.69 / \lambda \) years is required.

11. Empirical evidence of the model.
The expression for output growth rate:

\[
\frac{\dot{Y}(t)}{Y(t)} = \alpha_K(t) \frac{\dot{K}(t)}{K(t)} + \alpha_L(t) \frac{\dot{L}(t)}{L(t)} + R(t), \quad (14)
\]

The expression for output growth rate per worker:

\[
\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{L}(t)}{L(t)} = \alpha_K(t) \left( \frac{\dot{K}(t)}{K(t)} - \frac{\dot{L}(t)}{L(t)} \right) + R(t), \quad (15)
\]

where \( \alpha_L(t) \) – the elasticity of output with respect to labor at initial moment \( t \); \( \alpha_K(t) \) – the elasticity of output with respect to capital at initial moment \( t \) (\( \alpha_L(t)+\alpha_K(t) = 1 \)), they might be interpreted as shares of the labor and of the capital respectively in gross revenue. \( R(t) \) – «Solow residual» that explains the influence of knowledge (technological progress). In empirical works, \( R(t) \) explains 60-70% of growth!

11. Shortcomings of the model: a) an exogenous character of many inputs, such as saving and depreciation rates; b) the key factor of growth, i.e. technological progress, remains «a thing in itself» and unaccountable; c) the model does not take into account significant output factors such as land, natural resources, pollution, social institutions and so on.

### 4.2. Alternative Growth Models

1. The Lucas Model (AK model) – endogenous-growth model with two types of capital (physical and human). It is based on the Cobb-Douglas production function and presented as:

\[
Y_t = A K_t^\alpha H_t^{1-\alpha},
\]

where \( A \) - technological parameter (total factors of productivity), \( H \) - level of the human capital per unit of representative economic agent. \( I_t^p \) is investment of physical capital at the moment \( t \); \( I_t^h \) is investment of human capital at the moment \( t \). And \( \delta \) is the rate of depreciation for both capitals. Changes in the capital: \( \dot{K} = I^k - \delta \times K \) and \( \dot{K} = I^h - \delta K \). It is assumed that physical and human capital are perfect complementary goods. The equilibrium condition is: \( MP_k = MP_h \). After mathematical transformations, we receive:

\[
\frac{H_t}{K_t} = \frac{1-\alpha}{\alpha}.
\]

So the production function takes the form: \( Y = AK \), where \( A = \tilde{A} \left( \frac{1-\alpha}{\alpha} \right)^{1-\alpha} \times L^{(1-\alpha)} \) is the marginal as well as average productivity of capital.

Properties of the Lucas model: a) constant marginal productivity of the capital; b) equal rates of changes of the main variables: \( \frac{\dot{y}}{y} = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = sA - \delta \); c) convergence effect is absent (it is considered as a shortcoming of the model).

2. Ramsey-Kass-Koopmans model.
3. Model with overlapping generations.
4.3. Problems

**Problem 1.** Which parameters of the Solow growth model \((s, \delta, n, g)\) affect the level of output per worker and which have a growth effect?

**Problem 2.** Why rates of economic growth in developing countries are often higher than it in developed countries? How can you explain this phenomenon on the basis of the Solow Growth model? Under what conditions growth of output per capita in some poor country will be higher than it in a rich country?

**Problem 3.** \((\text{Solow growth model})\). Suppose some country with the production function \(Y = K^{1/2} \times (AL)^{1/2}\), and other parameters of the economy are: \(n = 2\%\), \(g = 6\%\), \(\delta = 7\%\), \(s = 0.45\).

Determine: a) the steady and the golden levels of capital per unit of effective labor \((k^* = ? k^{**} = ?)\); b) speed of convergence and half-life of it.

*Answer:* a) \(k^* = 9\), \(k^{**} = 11.1\); b) \(\lambda = 7.5\%\), \(t^* = 9.2\).

**Problem 4.** Suppose some country with production function like \(Y = K^{2/3} \times (AL)^{1/3}\), and other parameters of economy are: \(n = 1\%\), \(g_1 = 4\%\), \(g_2 = 6\%\), \(\delta = 5\%\), \(s = 0.30\).

Determine: a) the steady and the golden level of capital per unit of effective labor \((k^* = ? k^{**} = ?)\); b) speed of convergence and half-life of it.

*Answer:* a) \(k^* = 27\), \(k^{**} = 15.625\); b) \(\lambda = 4\%\), \(t^* \approx 17.3\).

**Problem 5.** Look at the figures 4.3 and 4.4 and answer the following questions:

1. Why does an average rate of economic growth differ steadily for groups of countries presented in Figure 4?

2. What do you think about economic fluctuations in the globalized world?

3. How did the economic crisis of 2008-2009 influence economic performances in different countries? Why was the fall of growth rates in the Commonwealth of Independent Countries more sizable than the world average? Why did the decrease of growth rates in the Developing Asia prove to be less than the world average?

![Figure 4.3. Gross Domestic Product in constant prices, groups of countries, percent changes](image-url)
6. Analyze information provided by the International Monetary Fund (URL: http://www.imf.org/external/pubs/ft/weo/2011/01/weodata/weoselgr.aspx) and examine economic processes in your country. Build a time series and relevant graphs for such economic variables as GDP at constant prices percentage change, output gap in percentage of potential GDP, investment percentage change, import and export volumes percentage change, unemployment rate, etc. Define their correlations and explain the results. Examine the dynamics of the above-mentioned parameters in your country.
UNIT 5. Business cycles and their models

5.1. Main propositions of the economic fluctuations theory

1. Business (economic) cycle consists of economy-wide fluctuations of the real GDP and economic activity around a long-term growth trend that last over several months or years. Its increasing wave includes: recovery (below the trend), expansion (above the trend) and boom, or peak (the highest point of the cycle). Its diminishing wave includes: recession (above the trend), depression (below the trend) and bottom (the lowest point of the cycle).

2. The variables (parameters) in the business cycle may be: procyclical, countercyclical, and acyclical; leading, lagging, and coincident.

3. Main types of cycles according to their periodicity (the first four are from the classification by Joseph Alois Schumpeter):
   - the Kitchin inventory cycle of 3–5 years (after Joseph Kitchin);
   - the Juglar fixed investment cycle of 7–11 years (often identified as 'the' business cycle);
   - the Kuznets infrastructural investment cycle of 15–25 years (after Simon Kuznets also called building cycle);
   - the Kondratiev wave or long technological cycle of 45–60 years (after Nikolai Kondratiev);
   - the Forrester energy supply and used materials cycles of 200 years (after Jay Wright Forrester);
   - the Toffler civilization cycles of 1000-2000 years (after Alvin Toffler).

4. Main theories of economic cyclicality: endogenous vs. exogenous.
   - Overinvestment theory (Thomas Robert Malthus);
   - Underconsumption theory (Jean Charles Léonard Simonde de Sismondi);
   - Outside factors theory, or sunspot theory (William Stanley Jevons, Henry Ludwell Moore);
   - Psychological theories (William Stanley Jevons, John Maynard Keynes);
   - The Marxist theory and the Goodwin model (after Richard M. Goodwin) ;
   - Innovative theory of economic development (Joseph Alois Schumpeter);
   - New Keynesian theories (John R. Hicks and Paul A. Samuelson);
   - Monetarist theory of business cycle (Milton Friedman and Edmund Phelps);

---

• Real business cycle theory (Finn E. Kydland and Edward C. Prescott);
• Politically-based business cycle theories (William Nordhaus).

5.2. Models of Economic Fluctuations

1. The Samuelson-Hicks multiplier-accelerator model

Main model. Investment multiplier: \( k = \frac{1}{1 - c_y} \), where \( c_y \) - marginal propensity to consume. Accelerator of investment: \( \nu = \frac{I_t}{(Y_{t-1} - Y_{t-2})} \).

Earned income (\( Y \)) in closed economy is spent to:
- consumption: \( C_t = C_a + c_y \times Y_{t-1} \), where \( C_a \) - autonomous consumption, that does not depend on income, because it depends on other factors;
- investment: \( I_t = I_a + \nu \times (Y_{t-1} - Y_{t-2}) \), where \( I_a \) - autonomous investment;
- autonomous government purchases: \( G_t = G_a \).

Put \( C_a + I_a + G_a = A_a \) - the sum of autonomous expenditures. So we get the income function in dynamic form: \( Y_t = C_t + I_t + G_t = A_a + (c_y + \nu) \times Y_{t-1} - \nu \times Y_{t-2} \).

When \( A_a \) is constant, income will attain some invariable level: \( \ddot{Y} = \frac{A_a}{1 - c_y} \). So an income function takes the form: \( Y_t = (1 - c_y) \times \ddot{Y} + (c_y + \nu) \times Y_{t-1} - \nu \times Y_{t-2} \).

When \( A_a \) changes, \( Y_t \) will also change gradually approaching its new steady level \( \ddot{Y} \). We can express deviation from it for each period of time: \( \Delta Y_t = Y_t - \dot{Y} \); \( \Delta Y_{t-1} = Y_{t-1} - \dot{Y} \); \( \Delta Y_{t-2} = Y_{t-2} - \dot{Y} \).

After substitution of \( Y_t \), \( Y_{t-1} \) and \( Y_{t-2} \) we receive main equation of the model: \( \Delta Y_t = (c_y + \nu) \times \Delta Y_{t-1} - \nu \times \Delta Y_{t-2} \).

By using the finite difference method for solving the differential equation, we can determine the value of the Discriminant: \( d = b^2 - 4ac \rightarrow \)

\[ d = (c_y + \nu)^2 - 4 \times \nu \]

Figure 5.2 represents the function for \( d = 0 \): \( c_y = -\nu + 2\sqrt{\nu} \).
Figure 5.2. Function \(d=0\) and the areas of monotonic and oscillatory changes

When \(d \geq 0\), alteration of \(Y_t\) will be monotonic.
When \(d < 0\), alteration of \(Y_t\) will be oscillatory.
When \(\nu < 1\), \(Y_t\) approaches the new stable level.
When \(\nu > 1\), \(Y_t\) deviates from the new stable level all the more.
When \(\nu = 1\), \(Y_t\) oscillates around \(\bar{Y}\) with a constant amplitude.

<table>
<thead>
<tr>
<th>Types of (Y_t) alteration relative to (\bar{Y})</th>
<th>(\nu &lt; 1)</th>
<th>(\nu = 1)</th>
<th>(\nu &gt; 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d \geq 0)</td>
<td>Monotonic convergence</td>
<td>(\bar{Y})</td>
<td>Monotonic divergence</td>
</tr>
<tr>
<td></td>
<td>(d &lt; 0)</td>
<td>Convergent oscillations</td>
<td>(\bar{Y})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Oscillations of a constant amplitude</td>
<td>(\bar{Y})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Divergent oscillations</td>
<td>(\bar{Y})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fluctuations would be bounded above and below!</td>
<td></td>
</tr>
</tbody>
</table>

2. Model with additional constraints. To bring the Samuelson-Hicks model closer to reality, it is necessary to constrain the upper and lower boundaries for \(Y_t\), when it changes in the zone III or IV (see Figure 5.2).
The upper boundary of output fluctuations coincides with the level of potential income \((Y_f)\). Therefore we have the first constraint: \(Y_f = \text{Min}\{(C_y + I_y + G_y + NX_y); Y_f \}\).

The lower boundary of output fluctuations is determined with the lowest negative level of induced investment equal to the yearly depreciation \((D)\). So we receive the second constraint: \(I_{ind} = \text{Max}\{(\nu \times (Y_{t-1} - Y_{t-2})\); D \}\).

As a result of these built-in constraints, the economy with \(\nu > 1\) will always turn into constant amplitude oscillations, independently of positive or negative value of the Discriminant (III or IV zone).

**Model with exogenous growth of autonomous expenditures.** Let the population grow annually at the rate \(n\), and therefore autonomous expenditures grow at the same rate.

So we receive:

- income function in the dynamic form: \(Y_t = A_{a0} \times (1 + n)^t + (c_y + \nu) \times Y_{t-1} - \nu \times Y_{t-2}\);

- dynamics of the steady level income: \(\tilde{Y}_t = (1 + n) \times Y_{t-1}\), after alteration it takes the form:

\[
\tilde{Y}_t = \frac{1}{1 - c_y + \nu + \nu \times (1 + \nu)^t} \times A_{a0} (1 + n)^t, \quad \text{where}
\]

\[
\frac{1}{1 - c_y + \nu + \nu \times (1 + \nu)^2} \quad \text{- the “supermultiplier” after Hicks;}
\]

- the upper boundary of the model is presented in the dynamic form: \(Y_{f_2} = Y_{f_1} (1 + n) = Y_{f_0} (1 + n)^t\);

- the lower boundary of the model is also presented in the dynamic form:

\[
\begin{align*}
D_t &= D_{t-1} (1 + n) = D_0 (1 + n)^t = -I_{ind0} \\
Y_t &= (A_{a0} - D_0) \times (1 + n)^t + c_y \times Y_{t-1} \\
Y_{t_{\text{min}}} &= \frac{(A_{a0} - D_0) (1 + n)^t}{1 - c_y/(1 + n)}.
\end{align*}
\]

As a result, income fluctuations get an inclined corridor (Figure 5.3).

**3. The Tewes model** supplements Samuelson-Hicks model with money market equilibrium and demonstrates the influence of monetary policy on cyclical fluctuations.

**Money demand:** \(L_t = l_y \times Y_{t-1} + l_i \times (i_{\text{max}} - i_t)\), where \(i_t\) - interest rate at the period \(t\); parameters of the model: \(l_y\) - money demand sensitivity of income change, and \(l_i\) - money demand sensitivity of interest rate.

**Money market equilibrium** with stable money supply \((M)\) and constant price level \((P=1)\):

\[
M = l_y \times Y_{t-1} + l_i \times (i_{\text{max}} - i_t) \rightarrow i_t = \frac{l_y}{l_i} \times Y_{t-1} - \frac{M - l_i \times i_{\text{max}}}{l_i}, \quad \text{and} \quad i_{t-1} = \frac{l_y}{l_i} \times Y_{t-2} - \frac{M - l_i \times i_{\text{max}}}{l_i} \quad \text{etc.}
\]

One of new prerequisites of the model is that current investment is sensitive to the previous period interest rate, so the investment function takes the form: \(I_t = I_a + \nu \times (Y_{t-1} - Y_{t-2}) - \gamma \times i_{t-1}\).

Assuming the total income is equal to total expenditures in closed economy, we obtain:

\[
Y_t = [C_y + c_y \times Y_{t-1}] + [I_y + \nu \times (Y_{t-1} - Y_{t-2}) - \gamma \times i_{t-1}] + G_y.
\]
After $i_{t-1}$ substitution in this expression, we receive the main equation of the model, that reflects alteration of actual income in dynamics: 

$$Y_t = B + (c_Y + v) \times Y_{t-1} - (v + \lambda) \times Y_{t-2},$$

where $B = A_u - \frac{M - \int_i^{i_{\text{max}}} l_i}{l_i}$ is absolute term of equation, and $\lambda = \gamma \times \frac{Y_t}{l_i}$ - is sensitivity ratio.

The stable level of $Y_t$ can be defined as: 

$$\bar{Y} = \frac{B}{1 - c_Y + \lambda}.$$

Discriminant of this equation: 

$$d = (c_Y + v)^2 - 4 \times (v + \lambda)$$

means:

- when $d \geq 0$, alteration of $Y_t$ will be monotonic;
- when $d < 0$, alteration of $Y_t$ will be oscillatory.

When $v + \lambda < 1$, the new equilibrium is stable.
When $v + \lambda > 1$, the new equilibrium is unstable.

When $v + \lambda = 1$, $Y_t$ oscillates around $\bar{Y}$ with a constant amplitude.

For $d = 0$ we receive: $c_Y = -v + 2\sqrt{v + \lambda}$ (Figure 5.4), which is higher than $c_Y = -v + 2\sqrt{v}$ – which is the $d = 0$ function for Samuelson-Hicks sample model.

Figure 5.4. The function $d=0$ for Tewes model in comparison with Samuelson-Hicks model, and the zones of different type of output changes

As a result, the zone of monotonous changes diminishes, and the zone of oscillatory changes extends. Because of the shift the top of the graph to the left, the zone of stable equilibrium is reduced and the zone of unstable equilibrium is increased. The zone of monotonous convergence becomes unachievable at all.

When the Central Bank conducts an active anti-cyclical monetary policy, money supply is presented in the form: $M = l_i \times i_{\text{max}} + \alpha \times Y_{t-1} + \beta \times i_t$.

Money market equilibrium in this case is: 

$$l_t \times Y_{t-1} + l_t \times (i_{\text{max}} - i_t) = l_t \times i_{\text{max}} + \alpha \times Y_{t-1} + \beta \times i_t$$

$$\rightarrow (l_t - \alpha) \times Y_{t-1} = (l_t + \beta) \times i_t \rightarrow i_{t-1} = \frac{l_t - \alpha}{l_t + \beta} \times Y_{t-2}.$$
Put \( y \times \frac{\alpha - l_y}{\beta + l_y} = h \). Obtain income in dynamics: \( Y_t = A_u + (c_y + \nu) \times Y_{t-1} - (\nu - h) \times Y_{t-2} \).

For this case the stable level of income: \( \ddot{Y} = \frac{1}{1-c_y-h} \times A_u \). Discriminant: \( d = (c_y + \nu)^2 - 4 \times (\nu - h) \). For the function \( d = 0 \) we receive: \( c_y = -\nu + 2\sqrt{\nu - h} \). When \( \alpha > l_y \), the \( d = 0 \) curve will be located lower along the scale of \( c_y \), relatively to the Samuelson-Hicks sample model, and the top of it will shift to the right. This means that the monotonic convergence admissible space will be enlarged and will become more probable. So by managing money supply function coefficients, the Central Bank can diminish the cyclicality and eliminate it altogether.

### 5.3. Problems

**Problem 1. (Samuelson-Hicks main model).** Suppose some economy in which the level of autonomous expenditures has increased from 100 to 200 units.

For different cases presented in Table 5.2, calculate the old and the new equilibrium level of income. Define the character of income alteration for each case.

<table>
<thead>
<tr>
<th>Case</th>
<th>Marginal propensity to consume ( c_y )</th>
<th>Accelerator ( \nu )</th>
<th>Discriminant ( d = (c_y + \nu)^2 - 4 \times \nu )</th>
<th>Type of ( Y ) changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.9</td>
<td>0.3</td>
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<tr>
<td>Case 2</td>
<td>0.7</td>
<td>0.8</td>
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<td></td>
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<tr>
<td>Case 3</td>
<td>0.7</td>
<td>1.0</td>
<td></td>
<td></td>
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<tr>
<td>Case 4</td>
<td>0.7</td>
<td>1.05</td>
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<tr>
<td>Case 5</td>
<td>0.8</td>
<td>2.4</td>
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</tbody>
</table>

Use the Excel program for calculating time series of income \( Y_t = (1-c_y) \times Y_{t-1} + (c_y + \nu) \times Y_{t-1} - \nu \times Y_{t-2} \) for 30 periods and draw the graphs for the function \( Y(t) \) to check the type of its alteration.

Draw the graph for function \( c_y = -\nu + 2\sqrt{\nu} \) (when \( d = 0 \)) and point the cases being examined on it.

**Problem 2. (Samuelson-Hicks model with additional constraints).** Suppose some economy, in which \( C_t = 100 + 0.75 \times Y_{t-1} \); \( I_t = 150 + 1.2 \times (Y_{t-1} - Y_{t-2}) \); \( G_{a0} = 50 \) units. The potential level of income is equal to 2500 units. Yearly depreciation of capital is 300 units.

Assume the government decided to support national economy and has increased internal purchases of goods and services by 100 units. Determine old and new equilibrium levels of output.

Using the algorithm presented in Table 5.3 and Excel calculation and graph drawing, define the character of real output changes in time. Calculate minimum and maximum boundary of output changes.
Table 5.3

<table>
<thead>
<tr>
<th>$t$</th>
<th>$Y_{t-1}$</th>
<th>$C_t = C_a + c_Y \times Y_{t-1}$</th>
<th>$I_a$</th>
<th>$I_{ind} = \nu \times (Y_{t-1} - Y_{t-2})$</th>
<th>$I_{ind}^* = \text{Max} {I_{ind}; -300}$</th>
<th>$G_a$</th>
<th>$Y_t = C_t + I_a + I_{ind}^* + G_a$</th>
<th>$Y_t^* =$ Min ${Y_t; 2500}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>150</td>
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<td></td>
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<td>50</td>
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</tr>
</tbody>
</table>

Answer. $\bar{Y}_1 = 1200$ units; $Y_1 = 1600$ units. Initially economy will demonstrate divergent oscillations, than it will come to a constant amplitude oscillations ($\approx 297.9; \approx 2259.6$).

Problem 3. Solve the problem 2, when in the economy being considered the value of accelerator ($\nu$) has increased up to 2.26.

Problem 4. (Samuelson-Hicks model with growth of population at a stable rate). Suppose some economy, where $c_Y = 0.6$ and $\nu = 1.25$. At initial period of time: $C_a = 200$, $I_a = 250$, and $G_a = 350$ units. The potential level of income at $t = 0$ is equal 2500 units. Depreciation of capital at $t = 0$ is equal to 300 units. The rate of population growth is 0.02 (i.e. $n = 2\%$) annually.

Using the algorithm of Table 5.4 and the Excel program, calculate time series of the autonomous expenditures, the depreciation level and the potential income for $t = 1,100$. Determine the minimum and maximum boundaries of output changes. Calculate time series of equilibrium and actual income for $t = 1,100$ and draw them on the graph as well as the upper and lower boundaries. Estimate the value of supermultiplicator.

Table 5.4

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous income $Y_{t-1}$</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autonomous expenditures $A_{nt} = A_{nt0} \times (1 + n)^t$</td>
<td></td>
<td>800</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Consumption from income $C_{Yt} = c_Y \times Y_{t-1}$</td>
<td></td>
<td></td>
<td>1200</td>
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<td></td>
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<tr>
<td>Induced investment $I_{indt} = \nu \times (Y_{t-1} - Y_{t-2})$</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td></td>
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<tr>
<td>Depreciation $D_t = D_0 (1 + n)^t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Induced investment - adjusted for depreciation $I_{indt}^* = \text{Max} {I_{indt}; -D_t}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
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<tr>
<td>Lower boundary of $Y_t$</td>
<td></td>
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</tr>
</tbody>
</table>
\[ Y_{\text{min}} = \frac{(A_{t0} - D_0)(1 + n)}{1 - c_y / (1 + n)} \]

**Actual income - calculated**
\[ Y_t = A_{tr} + C_{tr} + I_{indt} \]

**Potential income**
\[ Y_{f_t} = Y_{f0} (1 + n) - \text{upper boundary of } Y_t \]

**Actual income - corrected**
\[ Y^* = \text{Min} \{ Y_t; Y_{f_t} \} \]

**Equilibrium (steady level) income**
\[ Y_t = \frac{1}{1 - c_y + v + \frac{v}{(1 + v)^2}} \times A_{tr} \]

**Answer.** Supermultiplicator is equal to 2.58.

**Problem 5.** Suppose in the economy described in the problem 4, one of the following changes has taken place: a) the marginal propensity to consume out of income has decreased to \( c_y = 0.4 \); b) the rate of population growth has increased up to 4% \( (n = 0.04) \); c) the accelerator increased up to \( v = 2.5 \). Find a new solution for each case separately.

**Problem 6.** (Tewes model). Suppose some economy, in which \( C_t = 200 + 0.75 \times Y_{t-1} \); \( I_t = 100 + 0.6 \times (Y_{t-1} - Y_{t-2}) - 0.7 \times i_{t-1} \); \( G_a = 100 \) units. Demand for money is presented as follows: \( L_t = 0.8 \times Y_{t-1} + 1.6 \times (20 - i) \). Money supply is constant and equals 150 units.

Describe the algorithm of finding the \( Y(t) \) function. Using the Excel program, calculate the income time series and draw the \( Y(t) \) graph. Define the character of \( Y_t \) changes. Determine the stable level of income and the value of Discriminant.

**Answer.** Convergent oscillatory fluctuations. \( \bar{Y} = 543.75; \ d = -1.9775 \).

**Problem 7.** (Tewes model). Let’s imagine that in the economy described in the problem 6, the Central bank has begun to conduct an active policy from some period of time. Now the money supply function is determined as follows: \( M = 32 + 0 = 1.2 \times Y_{t-1} + 0.6 \times i_t \). Evaluate the multiplier and the Discriminant for dynamic income function. Calculate \( Y(t) \) time series and the stable level of \( Y_t \) and display them graphically. Define the character of income changes. Determine under which \( \alpha, \beta \) ratio the monotonic/oscillatory changes boundary appears to be crossed. What will be the value of \( h \) parameter and the level of multiplier in this case?
UNIT 6. Macroeconomic equilibrium and inflation

6.1. Main propositions of the inflation theory

1. *Opened inflation* is a rise in the general level of goods and commodities prices in an economy over a period of time.

*Repressed (suppressed) inflation* – inflation that is disguised by the government policy of prices, wages or exchange rate control or other interferences in the economy such as subsidies.

2. *Types of inflation*:
   - Creeping \((0 < \pi_t \leq 10\%)\).
   - Galloping \((20 < \pi_t \leq 200\%)\).
   - Hyperinflation \((50 \leq \pi_{monthly})\).

   Balanced vs. unbalanced. Expected vs. unexpected. Anticipated vs. non-anticipated.

   - a) demand-push inflation;
   - b) cost-push inflation;
   - c) inflationary spiral (the earliest model is «price-wage spiral»).

4. *Social costs of inflation*:
   - a) shoe leather cost;
   - b) menu cost;
   - c) relative-price variability and the misallocation of resources;
   - d) inflation-induced tax distortions;
   - e) «confusions and inconvenience».

5. *Distributive effects* of unpredicted raise in inflation:
   - a) distribution of incomes between capital and labor as factors of production;
   - b) distribution of incomes between persons with flexible (or indexed) and rigid (non-indexed) salaries;
   - c) distribution of incomes between creditors and debtors.

6. Positive effects of inflation:
   - a) labor market adjustment;
   - b) Central Bank maneuver with liquidity;
   - c) Mundell-Tobin effect.

7. Inflation impacts on the state budget condition:
   - a) Olivera-Tanzi effect – deterioration of real taxes proceeds (negative effect);
   - b) Patinkin effect – diminishing a real value of the part of budget expenditures that is nominally expressed (positive effect);
   - c) economic growth suppression (negative effect);
   - d) decrease in real cost of the public debt service (positive effect).

6.2. Models of Inflation

1. *The simple monetarist model*. Equilibrium of money market: \(\frac{M}{P} = L(Y, i)\), where \(M\) is the money stock, \(P\) - the price level, \(Y\) - the real income, \(i\) - the nominal interest rate. Real money supply is equal to real money demand. In the situation of excessive money supply, \(\frac{M}{P} > L(Y, i)\), when \(Y = const\), and \(i = const\), prices will rise.

   Neoclassical view: 1) economy is always in the conditions of full employment: \(Y = Y_f\); 2) Fisher identity takes place: \(i = r + \pi'\), so as \(\pi'\) and \(i\) grow at the same rate, and \(r = r = const\). Hereby: \(\frac{M}{L(Y, r + \pi')}\), and \(M\) and \(P\) change to equal percentage.

   Limitations of the model: a) in the situation of strengthening inflation expectation, \(\pi' \uparrow\), so \(L \downarrow\), thereby \(P\) will grow more rapidly than \(M\); b) excessive money supply for the first time will engender the «liquidity effect», therefore the short-term nominal and real rates may be temporarily decreased. It will cause the money demand increase, so prices will not grow completely in the short-run period.
2. Models of inflation based on the market equilibrium. In these models, inflation is considered as a result of dynamic interaction of aggregate demand and aggregate supply under the monetary or fiscal policy impact.

1. Aggregate supply dynamic function is based on:
   - Phillips curve: \( W_t = W_{t-1} \times \left[ 1 + \alpha \times \frac{N_t - N^*}{N^*} \right] \), where \( W_t \) - wage rate at the period \( t \), \( N_t \) - employment at the period \( t \), \( N^* \) - full employment, and \( \alpha \) - coefficient reflecting the sensitivity of wage rate in the current period to unemployment level in the previous period;
   - A. Okun’s interaction between output production and unemployment level: \( \frac{Y_t - Y_F}{Y_F} = -\gamma \times (u_t - u_n) \), where \( Y_t \) - actual output; \( Y_F \) - potential output; \( u_t \) - actual level of unemployment, \( u_n \) - natural level of unemployment, \( \gamma \) - coefficient of relating changes in unemployment to changes in output;
   - Cost-plus pricing: \( P_t = (1 + \lambda) \times \frac{N_t \times W_t}{Y_t} = (1 + \lambda) \times \tau \times W_t \), where \( \lambda \) - coefficient of markup to wage-cost, and \( \tau = \frac{N}{Y} \) - labor-output ratio of national income.

After alteration of them, taking into account that \( \frac{N^* - N_t}{N^*} = u_t - u_n \), we receive aggregate supply function in dynamic form: \( P_t = P_{t-1} \times \left[ 1 + \omega \times (Y_t - Y_F) \right] \), where \( \omega = \frac{\alpha}{\gamma \times Y_F} \) - coefficient reflecting reaction of the wage rate to output gap. The graph of this function represents the family of curves built up for different \( P_{t-1} \) level.

As the rate of inflation is defined as \( \pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \), so aggregate supply function would be presented in the form: \( \pi_t = \omega \times (Y_t - Y_F) \), or \( Y_t = Y_F + \frac{1}{\omega} \times \pi_t \).

Aggregate supply dynamic function with inflationary expectation in short-run: \( \pi_t = \omega \times (Y_t - Y_F) + \pi_t^e \), \( Y_t = Y_F + \frac{1}{\omega} \times (\pi_t - \pi_t^e) \). The graph of this function represents the family of curves built up for a different \( \pi_t^e \) level.

2. Aggregate demand dynamic function is deduced from IS – LM model, which establishes a joint equilibrium for goods and money markets:
   - IS function would be presented in the form: \( Y = \frac{A_y - \beta \times (i - \pi^e)}{\varsigma} \), where \( \varsigma = 1 - c_v = s_v + \tau \), and \( 1/\varsigma \) is the multiplier of autonomous expenditures, \( A_y = C_v + G_v + I_a \) is the sum of autonomous expenditures. Here the function of investment is \( I = I_a - \beta \times (i - \pi^e) \), where \( r \approx i - \pi^e \) is the real interest rate, and \( \pi^e \) is the expected level of inflation;
   - LM function would be presented in the form: \( M / P = l_\gamma \times Y + l_i \times (i_{\max} - i) \).

After expressing \( i \) from both IS and LM curve and equalization of them to each other, we receive the formula for equilibrium output: \( Y = a \times A_y + b \times (M / P) + c \times \pi^e - c \times i_{\max} \), where the coefficients of equation are determined as: \( a = \frac{l_i}{\varsigma \times l_i + \beta \times l_y} \), \( b = \frac{\beta}{\varsigma \times l_i + \beta \times l_y} \), \( c = \frac{\beta \times l_i}{\varsigma \times l_i + \beta \times l_y} \).
So we obtain the output in the dynamic form: 
\[ Y_t = a \times A_{t,t} + b \times (M_t / P_t) + c \times \pi_t - c \times i_{max}. \]

Alteration of output under the impact of monetary or fiscal impulse:
\[ \Delta Y_t = a \times \Delta A_{at} + b \times (M_t / P_t - M_{t-1} / P_{t-1}) + c \times (\pi_t - \pi_{t-1}) \]
\[ \implies Y_t = Y_{t-1} + a \times \Delta A_{at} + h \times (m_t - \pi_t) + c \times \Delta \pi_t \]

where \( m_t = \frac{M_t - M_{t-1}}{M_{t-1}} \) – rate of money supply change,
\[ h = b \times \frac{M_{t-1}}{P_t} \] – coefficient of the model made under the assumption that \( \frac{M_{t-1}}{P_t} \approx const. \]

3. \( AS - AD \) equilibrium model:
\[ Y_t = Y_e + \frac{1}{\omega} (\pi_t - \pi_e^c); \]
\[ Y_t = Y_{t-1} + a \times \Delta A_{at} + h \times (m_t - \pi_t) + c \times \Delta \pi_t. \]

4. Model of inflation in economics with static expectations (i.e. \( \pi_t^c = \pi_{t-1} \)):
\[ Y_t = Y_e + \frac{\Delta \pi_t}{\omega}; \]
\[ Y_t = Y_{t-1} + a \times \Delta A_{at} + h \times (m_t - \pi_t) + c \times \Delta \pi_{t-1}. \]

Suppose some economy at initial period of time is in the condition of full employment, i.e. \( Y_{t-1} = Y_{t-2} = \ldots = Y_e, \pi_{t-1} = \pi_{t-2} = \ldots = 0, m_{t-1} = 0, \Delta A_{at} = 0 \). Let from some period it experienced a monetary or fiscal shock. Examine the consequences of them separately.

**Consequences of an active monetary policy**

- single monetary impulse, \( m_t > 0; m_t = 0 \) for \( t = 2, \infty \) (\( \Delta A_{at} = 0 \)):
  
  the 1\textsuperscript{st} period:
  \[ Y_1 = Y_e + \frac{\pi_1}{\omega}; \]
  \[ Y_1 = Y_{t-1} + h \times (m_t - \pi_t). \]
  
  the 2\textsuperscript{nd} period:
  \[ Y_2 = Y_e + \frac{\pi_2 - \pi_1}{\omega}; \]
  \[ Y_2 = Y_{t-1} - h \times \pi_2 + c \times \pi_1. \]
  
  each other period:
  \[ Y_t = Y_e + \frac{\pi_t - \pi_{t-1}}{\omega}; \]
  \[ Y_t = Y_{t-1} - h \times \pi_t + c \times (\pi_{t-1} - \pi_{t-2}). \]

In this case in the long-run \( \pi_t \to 0, Y_t \to Y_e \), and convergent oscillations take place. Initial impact of monetary policy in the long-run will result in price level increase at the rate equal to money supply growth, but won’t influence the output, so AS curve during this period becomes vertical;

- permanent expansive monetary policy, \( m_t = \text{const} > 0 \) (\( \Delta A_{at} = 0 \)):
  
  \[ Y_t = Y_e + \frac{\Delta \pi_t}{\omega}; \]
  \[ Y_t = Y_{t-1} + h \times (m_t - \pi_t) + c \times \Delta \pi_{t-1}. \]

In this case \( \pi_t \to m_t \), \( Y_t \to Y_e \), and convergent oscillations take place as well. In this case the long-run AS curve is also vertical.
Indeed, according to the model, when \( Y_t > Y_F \), \( \pi_t \) increases. And when \( Y_t < Y_F \), \( \pi_t \) decreases. When \( m_t > \pi_t \), \( Y_t \) increases. But when \( m_t < \pi_t \), \( Y_t \) decreases. Figure 6.1(a) combines all types of equilibrium shift, and Figure 6.1(b) displays the general trajectory of its movement.

\[ \pi_t \]

\[ m_t \]

\[ Y_F \]

\[ Y_t \]

\[ \text{Figure 6.1. Direction of equilibrium change after monetary impulse} \]

\textbf{Consequences of an active fiscal policy}

- \textit{Single fiscal impulse}, \( \Delta A_{t+1} = A > 0 \); \( \Delta A_{t+1} = 0 \) for \( t \to \infty \) (while money supply grows at the constant rate: \( m_t = m = \text{const} > 0 \)):

the 1\(^{st}\) period:

\[
\begin{align*}
Y_t &= Y_F + \frac{(\pi_t - m)}{\omega}; \\
\pi_t &= \frac{m + \omega \times (a \times A + h \times m)}{1 + h \times \omega}; \\
Y_t &= Y_F + \frac{a \times A}{1 + h \times \omega};
\end{align*}
\]

each other period:

\[
\begin{align*}
Y_t &= Y_F + \frac{\pi_t - \pi_{t-1}}{\omega}; \\
Y_t &= \frac{\omega \times h \times m + (2 + c \times \omega) \times \pi_{t-1} - (1 + c \times \omega) \times \pi_{t-2}}{1 + h \times \omega}.
\end{align*}
\]

For this case in the long-run \( \pi_t \to m \), \( Y_t \to Y_F \), and \( Y_t \) as well as \( \pi_t \) oscillations are convergent, and the graphs presented in Figure 10 are valid, too.

- \textit{Permanent expansive fiscal policy}, \( \Delta A_{t+1} = A \) (and constant money supply growth rate, \( m_t = m = \text{const} > 0 \)):

\[
\begin{align*}
Y_t &= Y_F + \frac{\pi_t - \pi_{t-1}}{\omega}; \\
Y_t &= Y_{t-1} + h \times (m - \pi_t) + c \times (\pi_{t-1} - \pi_{t-2}); \\
\pi_t &= \frac{\omega \times (h \times m + a \times A) + (2 + c \times \omega) \times \pi_{t-1} - (1 + c \times \omega) \times \pi_{t-2}}{1 + h \times \omega}.
\end{align*}
\]

The steady level of inflation, meaning that \( \pi_t = \pi_{t-1} = \pi_{t-2} \), is \( \pi^* = m + \frac{a \times A}{h} \).
To define the character of $Y_i$ and $\pi_i$ changes, determine the Discriminant for $\Delta \pi_i$ dynamic function: $\Delta \pi_i = \frac{2 + c \times \omega}{1 + h \times \omega} \times \Delta \pi_{i-1} - \frac{1 + c \times \omega}{1 + h \times \omega} \times \Delta \pi_{i-2} \Rightarrow d = \left(\frac{2 + c \times \omega}{1 + h \times \omega}\right)^2 - 4 \times \left(\frac{1 + c \times \omega}{1 + h \times \omega}\right)$. The equilibrium is steady, when $\frac{1 + c \times \omega}{1 + h \times \omega} < 1 \Rightarrow c < h$. So far as $c = \frac{\beta \times l_i}{\varphi \times l_i + \beta \times l_Y}$, $h \equiv b \times \frac{M_{i-1}}{P}$, and $b = \frac{\beta}{\varphi \times l_i + \beta \times l_Y}$, we receive: $l_i < M_{i-1} / P$. Remember that LM function is $M / P = l_i \times Y + l_i \times (i_{\text{max}} - i)$, and both summands of the right-hand member of this equation are positive. Therefore the condition $l_i < M_{i-1} / P$ is always satisfied. So in the general case we have convergent oscillations for both endogenous parameters ($Y_i$ and $\pi_i$) of the model.

3. **Fiscal models of inflation (Seignorage and Inflation Tax).** In these models, money emission is considered as a method of financing the state budget deficit when alternative means are not available or their resources are exhausted.

*Seignorage* is the government revenue obtained from additional money supply. Real value of it: $S = \frac{M}{P} = \frac{M}{P} \times \frac{M}{P} = g_M \times \frac{M}{P}$, where $g_M$ is the rate of money growth.

*Inflation tax* is «the financial loss of value suffered by holders of cash and fixed-rate bonds, as well those on fixed income (not indexed to inflation), due to the effects of inflation»\(^3\). As opposed to seignorage, inflation tax means real depreciation of the money that circulated previously due to price rise. Real inflation tax: $IT = -\left.\frac{d(M / P)}{dt}\right|_{M=\text{const}} = \frac{M}{P^2} \times \frac{dP}{dt} = \frac{M}{P} \times \frac{\dot{P}}{P} = \pi \times \frac{M}{P}$, where $\pi$ is the rate of inflation. Inflation tax is equal to seignorage, when money does not affect the real production, and economic growth is absent.

*The model with seignorage* is based on the equality of real supply and real demand for money:

$$
\frac{M}{P} = L(i, Y) = L(r + \pi ^*, Y) = L(r + g_M, Y), \quad L_i < 0, \quad L_Y > 0.
$$

Therefore the first term of this equation is positive and the second term of it is negative and growing absolutely when $g_M \uparrow$. So we receive «inflation tax Laffer curve» (Figure 6.2). The maximum of this function ($S^*$) is achieved, when the elasticity of real money demand as to the rate of money change is equal to -1.

When government need for seignorage is fixed (G) and less than $S^*$, there are two equilibria in the model. In the case of adaptive expectations, the first one will be stable and the second one - unstable, so the system will come to the lower level of money growth and

inflation. In the case of rational expectations, the second equilibrium will be stable as opposed to the first one, so the system achieves a greater level of money growth and inflation.

4. **Dynamic Inconsistency of Low-Inflation Monetary Policy** (Kydland-Prescott model). «Dynamic inconsistency, or time inconsistency, describes a situation where a decision-maker’s preferences change over time in such a way that what is preferred at one point in time is inconsistent with what is preferred at another point in time»⁴.

Kydland-Prescott model is based on Lucas function, where rational expectations concerning inflation negatively affect aggregate supply, so the function of transformation takes the form: \( y = \bar{y} + b(\pi - \pi^e) \), \( b > 0 \), where \( \bar{y} \) is the level of output for perfectly flexible prices. The welfare loss function is: \( L(\pi) = \frac{1}{2}(y - y^*)^2 + \frac{1}{2}a(\pi - \pi^*)^2 \rightarrow \min \), where \( y^* \) and \( \pi^* \) are social optimaums of output and inflation respectively, and \( y^* > \bar{y}, \quad a > 0 \) (\( a \) reflects relative importance of inflation and output in welfare). After optimization it under limitation we receive the following solution:

\[
\pi = \pi^* + \frac{b}{a + b^2}(y^* - \bar{y}) = \pi^{EQ}. \quad \text{This yields: } y^* = \bar{y}, \text{ that is the output does not change at all when inflation increases.}
\]

Two approaches for solution of the dynamic inconsistency problem: a) importance of reputation for policymakers held «the wide planning horizon» (the Backus&Drifflill and Barro models of reputation); b) delegation of the control for policymakers to the third independent party (the Rogoff model).

### 6.3. Modern Peculiarities of Inflation

Types of modern inflation in the global context:

- **Agrarian inflation** (agflation) – leading growth of prices for food and other agrarian commodities.
- **Energy price inflation** – an advanced rise in prices for fuel and energy.
- **Assets prices inflation** – an excessive rise in prices for assets: financial instruments (stocks, bonds, derivatives etc.), real estate and capital goods. Usually it results in «financial bubbles».
  - Their appearance can be identified through the growth of the q-Tobin coefficient and the rise of «the financial depth» indicator by McKinsey Global Institute.
  - The peculiarities of Russian inflation:
    - - cyclical character of the average price level changes in long-run;
    - alteration of the cost-push inflation and the demand-push inflation;
    - particular cost-push inflation factors:
      - lack of competitive environments in some sectors of economy, the natural monopolies price-push behavior;
      - growth of the insurance tax payments from January 1, 2011;
      - devaluation of the ruble relative to the dollar (and later to the euro) influenced internal prices noticeably in the crisis 1998-1999, and had no perceptible effect in the crisis of 2008-2009.
    - particular demand-push inflation factors:

---

- inflows of the export revenues from petroleum, gas and other natural resources and conversion of foreign exchange into national money supply;
- favorable world market conditions as to above-named products contributed to internal income growth without adequate real output production rise;
- government expenditures changes fit into the politically-based business cycle.

Inflation rates in Russia in the long-run period of time are presented in Figure 6.3.

![Figure 6.3. The rates of inflation in the Russian Federation, %](image)

### 6.4. Problems

**Problem 1. (Models of inflation based on the market equilibriums).** Let’s imagine some closed economics with static expectations, where the potential output is \( Y_F = 1000 \) units, and the initial real money supply is \( \frac{M}{P} = 100 \) units. The real liquidity demand function is \( M / P = 0.05Y + 3 \times (20 - i) \). The sensitivity of wage rate to previous period unemployment level is \( \alpha = 0.5 \), and Okun’s coefficient: \( \gamma = 2.5 \). The function of consumption is presented as follows: \( C_t = 80 + 0.75Y_{t-1} \times (1 - 0.2) \), the function of investment: \( I_t = 180 - 4 \times (i - \pi^e) \), and the government purchases at zero period is \( G_{a0} = 200 \). Suppose that at zero period the economy is in the condition of steady equilibrium.

Determine:

1) The autonomous expenditures (\( A_{a0} \)) and the equilibrium interest rate (\( i_{E0} \)) at the initial period;

2) The form of the dynamic aggregate supply function is presented as \( Y_{tAS} = f(\pi_t; \pi_t^e) \);

3) The form of the dynamic aggregate demand function is presented as \( Y_{tAD} = f(Y_{t-1}; \Delta A_{a0}; m_t; \pi_t; \pi_t^e) \).

4) Let us assume that the Central Bank has increased the money supply by 20% just for the period \( t = 1 \) (\( m_1 = 0.2 \)). Using the Excel program, describe the algorithm of \( \pi_t \) and \( Y_t \) time series
calculation, draw the graphs of their time dependence and the graph of their interdependence \((\pi_s,Y_t)\) – the so-called inflationary spiral of monetary shock.

5) Let us assume that the Central Bank has launched the permanent expansion monetary policy and now increases the money supply by 20% annually \((m_t = 0.2)\). Using the Excel program, describe the algorithm of \(\pi_s\) and \(Y_t\) time series calculation, draw the graphs of their time dependence and the graph of their interdependence \((\pi_s,Y_t)\) – the inflationary spiral of monetary expansion.

6) Let us assume that the Government has increased the public purchases by 80 units just at the period \(t = 1\) \((\Delta G_{a1} = 80)\). The Central Bank continues to increase the money supply by 20% annually \((m_t = 0.2)\). Using the Excel program, describe the algorithm of \(\pi_s\) and \(Y_t\) time series calculation, draw the graphs of their time dependence and the graph of their interdependence \((\pi_s,Y_t)\) – the inflationary spiral of fiscal shock \((\pi_s,Y_t)\). What will be the steady level of inflation?

7) Let us assume that the Government has launched the permanent expansion fiscal policy and now increases the public purchases by 80 units annually. The Central Bank continues to increase money supply for 20% annually \((m_t = 0.2)\). Using the Excel program, describe the algorithm of \(\pi_s\) and \(Y_t\) time series calculation, draw the graphs of their time dependence and the graph of their interdependence \((\pi_s,Y_t)\) – the inflationary spiral of fiscal expansion \((\pi_s,Y_t)\). What will be the steady level of inflation in this case?

8) By using the developed algorithms of \(\pi_s\) and \(Y_t\) time series calculation, estimate the Discriminant value for each case. How does it depend on the value of the following parameters: \(\alpha\), \(\beta\), \(\gamma\), \(\zeta\), \(l_i\), \(l_Y\) ?

Answer: 1) \(i_{E0} = 15\); \(A_{a0} = 460\); 2) \(Y_t^{AS} = 1000 + 5000 \times (\pi_t - \pi_{t-1})\); 3) \(Y_t^{AD} = Y_{t-1} + 2.14 \times \Delta Y_t + 285.7 \times (m_t - \pi_t) + 8.57 \times \Delta \pi_t^s\); 6) \(\pi^* = 0.2\); 7) \(\pi^* = 0.8\).

**Problem 2.** How can you explain the leading rise of agricultural and energy prices in the world economy? By using the statistical data provided by the European commission (URL: http://epp.eurostat.ec.europa.eu/portal/page/portal/hicp/data/database), examine the prices indexes dynamics for separate goods and services included in the consumer basket. Estimate their relative changes as to all-terms harmonized CPI, and explain the result obtained on the basis of the theory of unbalanced inflation and branch markets determinants.

**Problem 3.** Analyze the figure 12 for Russian economy and explain the difference in inflation indexes relative changes. How their dynamics can be explained from the standpoint of «cost-pusher» and «demand-pusher» inflation theory? By using the national statistical data, construct the time series for the deflator, CPI and PPI in your country, draw relevant graphs and analyze divergence in their dynamics.
RECOMMENDED LITERATURE

Main literature:

Additional literature:
APPENDIX
(definitions are quoted from *Macroeconomics* by G. Mankiw)

**Effects of economic policy**

⇒ *Multiplier effect* – the additional shifts in aggregate demand that result when expansionary fiscal policy increases income and thereby increases consumer spending.

⇒ *Automatic stabilizers* – changes in fiscal policy that stimulate aggregate demand when the economy goes into a recession without policymakers having to take any deliberate action.

⇒ *Catch-up effect* – the property whereby countries that start off poor tend to grow more rapidly than countries that start off rich.

⇒ *Natural-rate hypothesis* – the claim that unemployment eventually returns to its normal, or natural, rate, regardless of the rate of inflation.

⇒ *Sacrifice ratio* – the number of percentage of annual output lost in the process of reducing inflation by 1 percentage point.

⇒ *Rational expectations* – the theory according to which people optimally use all the information they have, including information about government policies, when forecasting the future.

**Main problems of economic policy**

Five debates over macroeconomic policy (by G. Mankiw):
1. Consider whether policymakers should try to stabilize the economy.
2. Consider whether monetary policy should be made by rule rather than by discretion.
3. Consider whether the central bank should aim for zero inflation.
4. Consider whether the government should balance its budget.
5. Consider whether the tax laws should be reformed to encourage saving.

**Glossary**

⇒ *Nominal GDP* – the production of goods and services valued at current prices.

⇒ *Real GDP* – the production of goods and services valued at constant prices.

⇒ *GDP deflator* – a measure of the price level calculated as the ratio of nominal GDP to real GDP times 100.

⇒ *Consumer price index* (CPI) – a measure of the overall cost of the goods and services bought by a typical consumer. (The basket of goods and services).

⇒ *Inflation rate* – the percentage change in the price index from the preceding period.

⇒ *Producer price index* – a measure of the cost of a basket of goods and services bought by firms.

⇒ *Nominal interest rate* – the interest rate as usually reported without a correction for the effects of inflation.

⇒ *Real interest rate* – the interest rate corrected for the effects of inflation.

⇒ *Productivity* – the amount of goods and services produced from each hour of a worker’s time.

⇒ *Physical capital* – the stock of equipment and structures that are used to produce goods and services.

⇒ *Human capital* – the knowledge and skills that workers acquire through education, training and experience.
Natural resources – the inputs into the production of goods and services that are provided by nature, such as land, rivers, and mineral deposits.

Technological knowledge – society’s understanding of the best ways to produce goods and services.

Financial markets – financial institutions through which savers can directly provide funds to borrowers.

Bond – a certificate of indebtedness.

Stock – a claim to partial ownership in a firm.

Financial intermediaries – financial institutions through which savers can indirectly provide funds to borrowers.

Mutual fund – an institution that sells shares to the public and uses the proceeds to buy a portfolio of stocks and bonds.

Market for loanable funds – the market in which those who want to save supply funds those who want to borrow to invest demand funds.

Labor force – the total number of workers, including both the employed and the unemployed.

Unemployment rate – the percentage of the labor force that is unemployed.

Labor-force participation rate – the percentage of the adult population that is in the labor force.

Natural rate of unemployment – the normal rate of unemployment around which the unemployment rates fluctuate.

Cyclical unemployment – the deviation of unemployment from its natural level.

Discouraged workers – individuals who like to work but have given up looking for a job.

Union – a worker association that bargains with employers over wages and working conditions.

Collective bargaining – the process by which unions and firms agree on the terms of employment.

Unemployment insurance – a government program that partially protects worker’s incomes when they become unemployed.

Money – the set of assets in an economy that people regularly use to buy goods and services from other people.

Medium of exchange – an item that buyers give to sellers when they want to purchase goods and services.

Unit of account – the yardstick people use to post prices and record debts.

Store of value – an item that people can use to transfer purchasing power from the present to the future.

Liquidity – the ease with which an asset can be converted into the economy’s medium of exchange.

Commodity money – money that takes the form of commodity with intrinsic value.

Fiat money – money without intrinsic value that is used as money because of government decree.

Currency – the paper bills and coins in the hands of the public.

Demand deposits – balances in bank accounts that depositors can access on demand by writing a check.

Central bank – an institution designed to oversee the banking system and regulate the quantity of money in the economy.

Money supply – the quantity of money available in the economy.

Monetary policy – the setting of the money supply by policymakers in the central bank.
Reserves – deposits that banks have received but have not loaned out.
Fractional-reserve banking – a banking system in which banks hold only a fraction of deposits as reserves.
Reserve ratio – the fraction of deposits the banks hold as reserves.
Money multiplier – the amount of money the banking system generates with each dollar of reserves.
Open-market operations – the purchase and sale of the government bonds by the central bank.
Reserve requirements – regulations on the minimum amount of reserves that banks must hold against deposits.
Discount rate – the interest rate on the loans that the central bank makes to banks.
Quantity theory of money – a theory asserting that the quantity of money available determines the price level and the growth rate in the quantity of money available determines the inflation rate.
Nominal variables – variables measured in monetary units.
Real variables – variables measured in physical units.
Classical dichotomy – the theoretical separation of nominal and real variables.
Monetary neutrality – the proposition that changes in the money supply do not affect real variables.
Velocity of money – the rate at which money changes hands.
Fisher effect – the one-for-one adjustment of the nominal interest rate to the inflation rate.
Shoeleather costs – the resources wasted when inflation encourages people to reduce their money holdings.
Menu costs – the costs of changing prices.
Net exports – the value of a nation’s exports minus the value of its imports, also called the trade balance.
Net foreign investment – the purchase of foreign assets by domestic residents minus the purchase of domestic assets by foreigners.
Nominal exchange rate – the rate at which a person can trade the currency of one country for the currency of another.
Appreciation – an increase in the value of a currency as measured by the amount of a foreign currency it can buy.
Depreciation – a decrease in the value of a currency as measured by the amount of a foreign currency it can buy.
Real exchange rate – the rate at which a person can trade the goods and services of one country for the goods and services of another.
Purchasing-power parity – a theory of exchange rates whereby a unit of any given currency should be able to buy the same quantity of goods in all countries.
Aggregate-demand curve – a curve that shows the quantity of goods and services that households, firms, and the government want to buy at any price level.
Aggregate-supply curve – a curve that shows the quantity of goods and services that firms choose to produce and sell at any price level.
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МАКРОЭКОНОМИКА
(ПРОДВИНУТЫЙ УРОВЕНЬ)

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